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Level 2 Mathematics and Statistics, 2016

91267 Apply probability methods in solving problems

9.30 a.m. Thursday 24 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–15 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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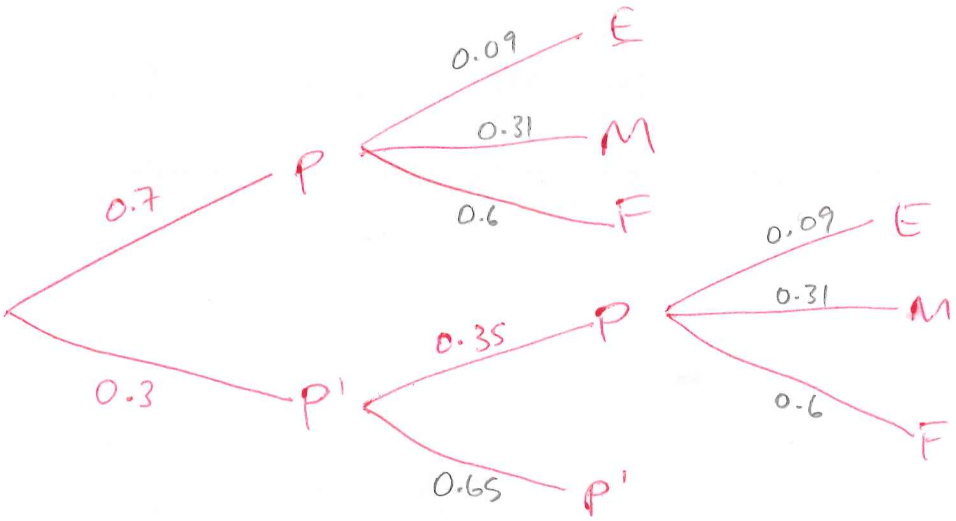
(a) The trees in Brookhaven apple orchard are harvested twice a year, as the apples ripen at different times.

Records from previous years have shown that during the first harvest, 70% of apples are picked, while the rest are left. At the second harvest, 35% of the remaining apples are picked, while the rest are not picked.

All the apples that are picked go to the packing shed. From each harvest, 9% are selected to be sent for export; 31% are sent for sale on the local market; the rest are sent to a factory to be made into sauce.



(i) Find the probability that a randomly selected apple will not be picked.

$0.3 \times 0.65 = 0.195$

(ii) What proportion of apples will be sent to the factory to be made into sauce?

(0.7×0.6) or $+$ $(0.3 \times 0.35 \times 0.6)$

$= 0.483$

(iii) If an apple is sold on the local market, then what is the probability that it will have been from the first harvest?

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$\frac{\text{Picked first \& market}}{\text{Market}} = \frac{0.7 \times 0.31}{(0.7 \times 0.31) + (0.3 \times 0.35 \times 0.31)} = \frac{0.217}{0.24955}$

$= 0.8697$

(iv) There are 120 apples in an export carton.

If 172 export cartons are produced, then how many apples were there in the total crop?

Total apples exported $= 120 \times 172$

$= 20640$

$P(\text{exported}) = (0.7 \times 0.09) + (0.3 \times 0.35 \times 0.09)$

$= 0.063 + 0.00945$

$= 0.07245$

$P(\text{exported}) \times \text{total number of apples} = 20640$

$0.07245 \times x = 20640$

$x = \frac{20640}{0.07245}$

$= 284,886.1284$

$\therefore$ 284,887 apples.

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(b) At the Pipsy Galore orchard only two varieties of apple, Jazz and Beauty, are grown.

Twice as many Jazz apples as Beauty apples are grown.

$\frac{2}{3}$ $\frac{1}{3}$

Table 1 below shows the proportions of each variety of apple that have been picked after both harvests have been completed.

Table 1

	Picked	**Not picked**
Jazz	0.85	0.15
Beauty	0.95	0.05

Table 2 shows the proportion of **picked** apples for each variety that are exported, sold locally, or sent to the factory.

Table 2

	Export	**Local**	**Factory**
Jazz	0.12	0.58	0.3
Beauty	0.15	0.7	0.15

(i) What is the probability that an apple selected at random from the **total crop** will be of the Jazz variety and will be sent to the factory?

P(Jazz & Factory) = $\frac{2}{3} \times 0.85 \times 0.3$

= 0.17

(ii) A minimum of 120 of the 294 export cartons produced at this orchard must be of the Beauty variety.

By showing calculations to support your answer, determine if this condition can be met.

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$$\frac{\text{Beauty export}}{\text{Export}} = \frac{\frac{1}{3} \times 0.95 \times 0.15}{\left(\frac{1}{3} \times 0.95 \times 0.15\right) + \left(\frac{2}{3} \times 0.85 \times 0.12\right)}$$

$$= \frac{0.0475}{(0.0475 + 0.068)}$$

$$= 0.4113$$

$$294 \times 0.4113 = 120.9222$$

$\therefore$ The conditions are able to be met.

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QUESTION TWO

Crisp Orchard has two blocks of land where apples are grown. Apples from the larger block are grown by conventional methods, while in the other block they are grown organically.

A random sample of 1200 apples is taken over both blocks and tested for disease. The results are summarised in Table 3 below.

Table 3

	Conventional	Organic	Total
Diseased	122	58	180
Not diseased	518	502	1020
Total	640	560	1200

(a) (i) What proportion of apples in the sample were diseased?

$\frac{180}{1200} = 0.15$

(ii) What proportion of the diseased apples were conventionally grown?

$\frac{122}{180} = 0.67\dot{7}$

(iii) If there was a total of 171 000 organic apples grown, then based on this sample, how many apples would be expected to be grown conventionally and be diseased?

total number of apples

$\frac{560}{1200} \times x = 171\,000$

$x = 366428.5714$

$\frac{122}{1200} \times 366{,}428.5714 = 37253.57143$

$\therefore$ expect 37,253 or 37,254 apples to be grown conventionally and diseased.

(iv) It is claimed that apples that are conventionally grown are at least twice as likely to be diseased as apples that are grown organically.

State whether you agree with this claim, showing full calculations to support your answer.

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$P(\text{Conv } \& \text{ diseased}) = \frac{122}{640} = 0.190625$

$P(\text{Org } \& \text{ dis}) = \frac{58}{560} = 0.1036$

$RR = \frac{0.1906}{0.1036}$

$= 1.8$

$\therefore$ conventionally grown apples are not twice as likely to be diseased as those grown organically, they are 1.8 times as likely

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(b) Jazz and Beauty varieties are grown in both blocks.

From the same sample of 1200 apples in part (a), 890 apples were of the Jazz variety.

It was also found that 182 of the Beauty variety were not diseased.

Table 3 from part (a) is repeated here to help you answer the questions that follow.

Table 3

	Conventional	**Organic**	**Total**
Diseased	122	58	180
Not diseased	518	502	1020
Total	640	560	1200

(i) What proportion of apples in the sample were of the Jazz variety and diseased?

	D	D'	
J	52	838	890
B	128	182	310
	180	1020	1200

$$\frac{52}{1200} = 0.043\dot{3}$$

(ii) It is claimed that apples are more likely to be diseased depending on their variety than whether they have been conventionally or organically grown.

Can this claim be supported?

Give calculations of absolute and relative risks to support your answer.

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Risk jazz^{dis} $= \frac{52}{890} = 0.0584$

Risk Beauty dis $= \frac{128}{310} = 0.4129$

$RR = \frac{0.4129}{0.0584}$

$= 7.067$

This is more than the 1.84 from the two types of growth. This implies that variety is more likely to explain disease.

QUESTION THREE

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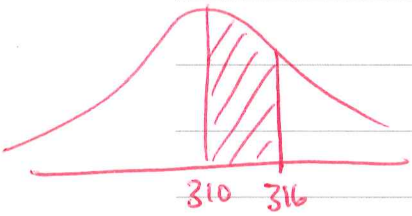
Apples are sent to a factory to be made into sauce, then bottled.

Testing has shown that, when the bottling machine is operating correctly, the weight of sauce dispensed into a bottle can be taken to be normally distributed, with mean 310 grams and standard deviation 4.5 grams.

(a) (i) Find the probability that a randomly selected bottle will contain between 310 and 316 grams of sauce.



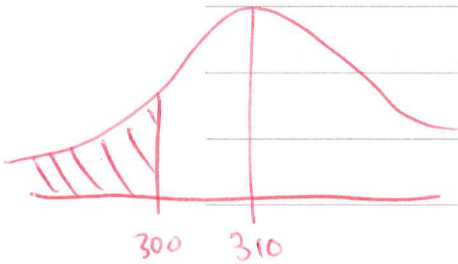
$$z = \frac{316-310}{4.5}$$

$$= 1.33\dot{3}$$

$$P(310 < x < 316) = 0.4087$$

(ii) The label on the sauce bottle states that the contents weigh 300 grams. Bottles that contain less than 300 grams are considered to be under-weight.

What percentage of bottles would be expected to be under-weight?



$$z = \frac{300-310}{4.5}$$

$$z = -2.222$$

$$P(x < 300) = 0.5 - 0.4869$$

$$= 0.0131$$

$$0.931 \times 100 = 1.31\ \%$$

We would expect around 1.31% to be underweight.

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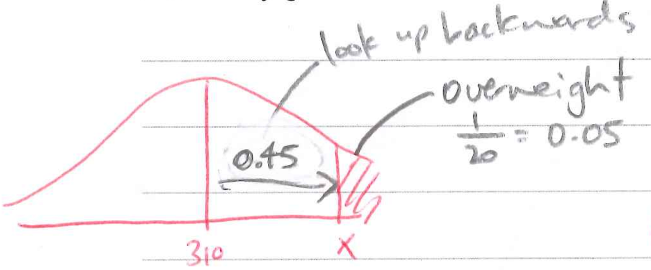
(iii) A quality control process identifies bottles of sauce that have contents that are over-weight.

When the machine is functioning correctly, no more than one bottle in 20 is over-weight.

If more than one in 20 bottles are over-weight, then the machine must be adjusted.

How many grams of sauce will a bottle contain before it is considered over-weight?



$z = \frac{X - 310}{4.5}$

$1.645 = \frac{X - 310}{4.5}$

$X = 317.4025$

$\therefore$ around 317.4g or larger would be considered over weight

(iv) The quality control process involves taking a random sample of three successive bottles and measuring the weights of their contents. If more than one of the bottles is found to be over-weight or under-weight, then the machine is checked for possible adjustment.

After taking a random sample, what is the probability that the machine is checked?

$P(\text{over}) + P(\text{under}) = 0.05 + 0.01313$

$= 0.06313$

P(over or under) and P(over or under) and P(over or under)

$0.06313 \times 0.6313 \times 0.6313$

$= 0.06313^3$

P(all 3 under or over) $= 0.000251$

any order (over or under) (fine)

P(2 are under or over, 1 is fine) $= 3 \times 0.06313^2 \times 0.9369$

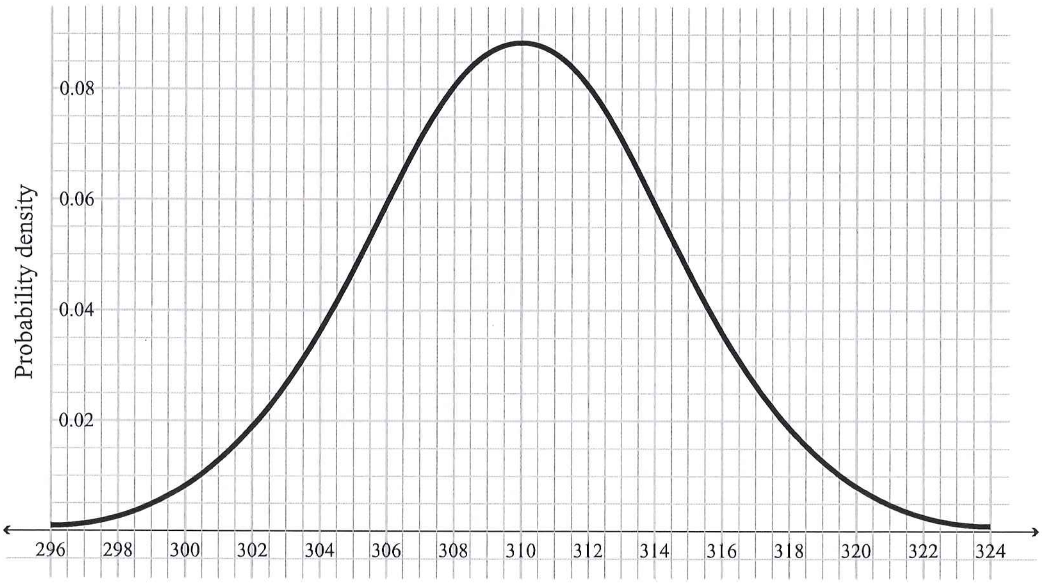
$= 0.0112$ (4dp)

Final Answer $= 0.000251 + 0.0112 = 0.0115$

(b) If the bottling machine was operating correctly, the weight of sauce dispensed into a bottle would have a probability distribution similar in shape to the one in Figure 1 below.

This graph has been corrected from that used in the examination.

Figure 1

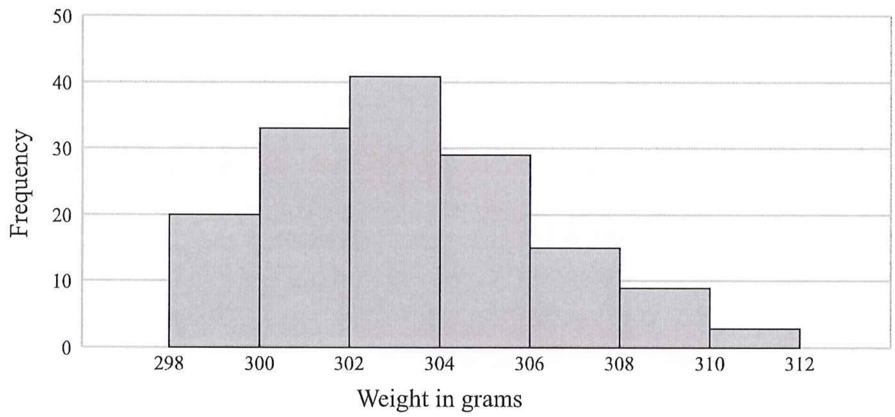


A new bottling machine is installed.

A random sample is taken of the contents of 150 bottles to test the new machine. The results are shown in the frequency histogram in Figure 2 below.

Figure 2

Sample Results



(i) What proportion of bottles in the sample had contents that were under-weight (i.e. the contents weighed less than 300 grams)?

$\frac{20}{150} = 0.1\dot{3}3\dot{3}$

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(ii) Compare the probability distribution with the frequency histogram that was obtained from the results of the sample.

In your answer, you should consider the shape, centre, and spread of both distributions and provide numerical evidence.

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Shape ① is bell shaped
② not symmetrical – skewed to right

Centre ① mean, median, mode all ≈310
② mode 302-304
median & mean likely to be smaller

Spread ① Range around 28
② Range around 14

Calculations ① $P(x > 310) = 0.5$
② $P(x > 310) = \frac{2}{150}$
$= 0.01333$

Considerably different

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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