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NZQA

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Level 2 Mathematics and Statistics, 2018

91267 Apply probability methods in solving problems

9.30 a.m. Wednesday 14 November 2018
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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Each day, temperatures are recorded every hour at various locations around New Zealand. The National Institute of Water and Atmospheric Research (NIWA) obtains the "daily temperature" by finding the mean of all the recorded temperatures in one 24-hour period at that location.

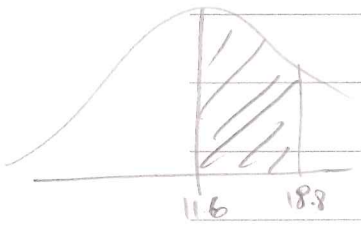
(a) Over the last seven years, the daily temperatures in Reefton have been found to be approximately normally distributed with a mean of 11.6°C and standard deviation of 4.8°C.

Use this approximation to answer the questions in parts (i) to (iv).

Working and/or diagrams must be shown. Correct answer(s) alone will generally limit grades to Achievement.

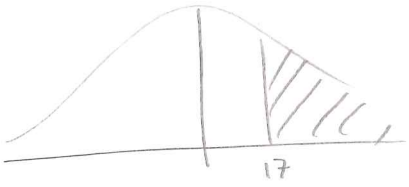
(i) Find the probability that a randomly chosen day in Reefton would have a daily temperature between 11.6°C and 18.8°C.

$\sigma = 4.8$



$P = 0.4332$

(ii) Find the probability that a randomly chosen day in Reefton would have a daily temperature over 17°C.



$P = 0.1303$

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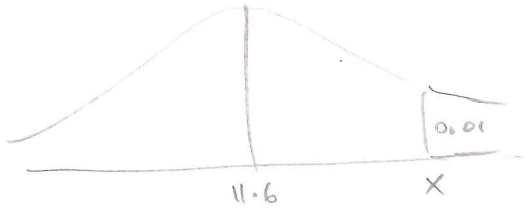
(iii) People in Reefton talk about the hottest 1% of days as being "scorchers".

What is the lowest possible daily temperature of a "scorcher" day?

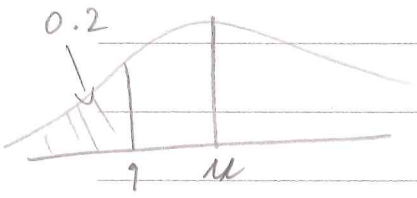


$X = 22.77$

(iv) Using this normal distribution approximation, the probability of a daily temperature lower than 9°C is 29.4% (3sf).

Suppose that temperatures in Reefton increased due to climate change, but the standard deviation remained the same.

What would the mean daily temperature for Reefton need to become to make this probability fall to 20.0% (3sf)?



$$z = \frac{x - \mu}{\sigma}$$

$$-0.841 = \frac{9 - \mu}{4.8}$$

$\mu = 13.0368$

(b) Show that, for **any** normal distribution, the inter-quartile range (IQR) is always greater than the standard deviation.

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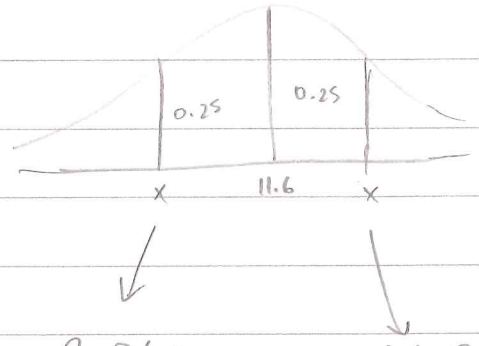

8.362 14.8375

IQR = 6.4755

σ = 4.8

Because 0.25 is has a z value of 0.674

0.674 × 2 = 1.348

∴ it is 1.348 stad deviations

QUESTION TWO

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(a) Matiu analysed Kaitaia's daily weather data from NIWA for the last seven years. He classified a day as "windy" if the mean wind speed was above 36 km/h, and "still" otherwise. He classified a day as "wet" if more than 2 mm of rain fell, and "dry" otherwise. He summarised his analysis in Table 1 below.

Table 1

Kaitaia	Wet	Dry	Total
Windy	553	1093	1646
Still	88	822	910
Total	641	1915	2556

Use Table 1 to answer the questions in parts (i) to (iii).

(i) What proportion of days over this time were windy?

$\frac{1646}{2556} = 0.644$

(ii) What proportion of wet days over this time were still?

$\frac{88}{641} = 0.1373$

(iii) Over these years, is it more likely to be wet on a windy day, or wet on a still day?
Support your answer with probability calculations.

Wet on windy $\frac{553}{1646} = 0.336$

Wet on still $\frac{88}{910} = 0.0967$

RR = 3.47 as likely to be wet on windy

(iv) A school in Kaitaia is planning its Athletics Day for February 2019. The Sports Co-ordinator uses the information in Table 1 to calculate the probability of the day being dry and still.

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Give one statistical reason why using Table 1 might not lead to a valid probability.

OLD data & some is 7 years old, conditions won't necessarily be the same.

Seasonal weather will play a part.

(b) Matiu wants to compare the weather in Kaitaia with the weather in Reefton, where his brother Tamati lives. He performs the same analysis of weather data for each town to obtain the summaries in Tables 1 and 2 below.

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Table 1 *(reprinted from page 5)*

Kaitaia	Wet	Dry	Total
Windy	553	1093	1646
Still	88	822	910
Total	641	1915	2556

Table 2

Reefton	Wet	Dry	Total
Windy	242	53	295
Still	641	1620	2261
Total	883	1673	2556

Matiu claims that it is 20 times more likely to be windy when it is dry in Kaitaia than it is in Reefton.

Using the information in Tables 1 and 2, explain whether Matiu's claim is correct or not.

K Windy when dry $\frac{1093}{1915} = 0.5708$

R $\frac{53}{1673} = 0.0317$

RR = 18.02

∴ not 20 times more likely more like 18 times.

QUESTION THREE

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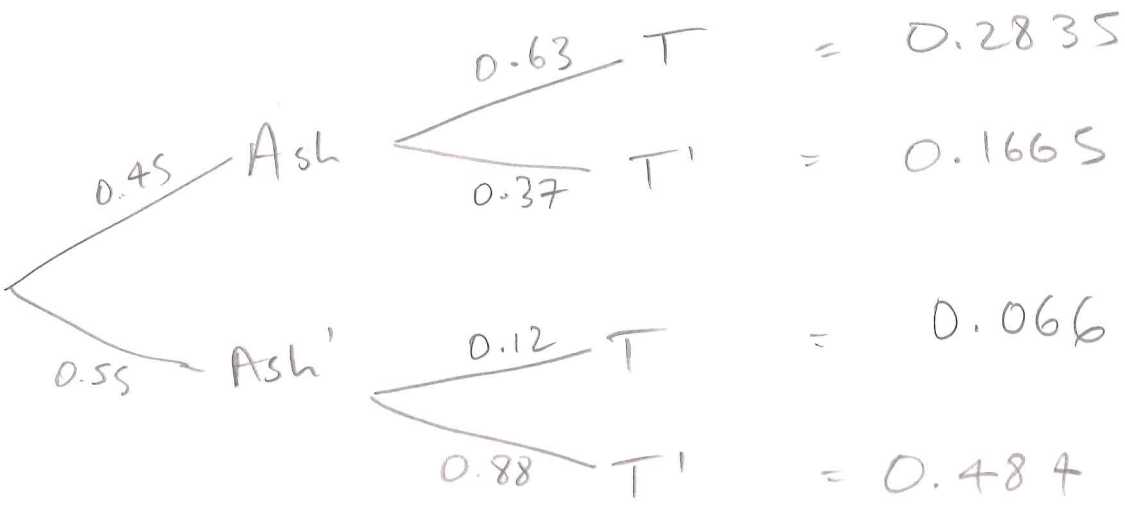
Nancy finds some data from NIWA on weather in Ashburton and in Timaru over the past seven years.

She analyses the data and finds that:

- It was wet on 45% of days in Ashburton.
- If it was wet in Ashburton, the probability that it was wet on the same day in Timaru was 63%.
- If it was dry in Ashburton, the probability that it was dry on the same day in Timaru was 88%.



(a) (i) Find the probability that it was dry in both Ashburton and Timaru on a randomly chosen day.

$0.55 \times 0.88 = 0.484$

(ii) Find the probability that, on a randomly chosen day, only one of the towns was wet.

$0.066 + 0.1665 = 0.2325$

(iii) If it was a dry day in Timaru, what is the probability that it was also dry in Ashburton on the same day?

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Dry in Ash and Tim $\rightarrow 0.484$

Dry in Tim $\rightarrow 0.1665 + 0.484$

$$\frac{0.484}{0.1665 + 0.484} = 0.7440$$

(b) Nancy is planning a one-day bicycle ride from Ashburton, through Timaru and then to Waimate. Nancy's grandfather says that, in his experience, the probability of it being dry in all three towns on the same day is "somewhere between 30 and 35%".

On the day of Nancy's ride, it is dry in both Ashburton and Timaru.

If Nancy's grandfather is correct, what is the smallest probability that it is a wet day in Waimate?

Ash Tim Wai

$0.55 \times 0.88 \times x = 0.3$

$\therefore x = 0.48$ ← W being dry

$0.55 \times 0.88 \times x = 0.35$

$x = 0.7231$ ← W being dry

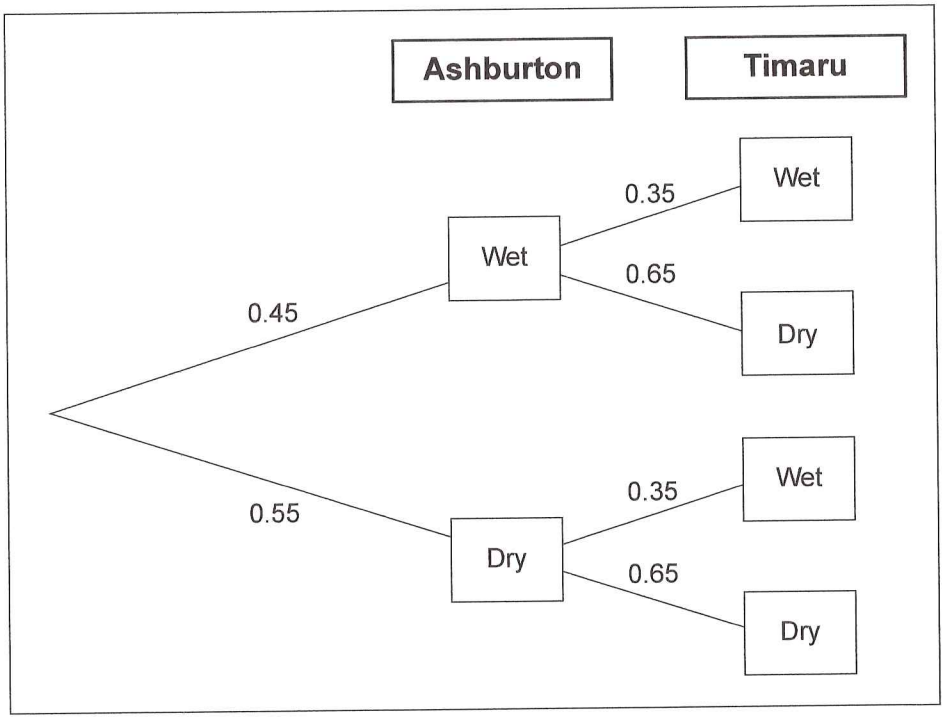
$1 - 0.7231 = 0.2769$

(c) Nancy's friend Teri uses the NIWA data for the past seven years to find out that:

- 45% of days in Ashburton were wet
- 35% of days in Timaru were wet.

Teri constructs the tree diagram in Figure 1 below.

Figure 1



0.3575

Figure 2

Adapted from: http://ortho.linz.govt.nz/nz_small_scale/si_1million.jpg

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Explain why Teri's tree diagram would **not** give a correct answer to the probability that it is dry in both towns on the same day.

As part of your justification, you could:

- compare your answer from part (a)(i) with the answer that Teri would get from her tree diagram
- consult the map of Canterbury in Figure 2.

Because 35% is prob T being Wet

Not P(Twet given Ash Wet)

Not conditional probabilities.

Climate data for this examination is adapted from 'CliFlo: NIWA's National Climate Database on the Web', http://cliflo.niwa.co.nz/, retrieved 10 November 2017.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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