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SAMPLE PAPER

NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 3 Mathematics and Statistics (Statistics)

91585 (3.13): Apply probability concepts in solving problems

Credits: Four

Check that you have completed ALL parts of the box at the top of this page.

You should answer ALL parts of ALL questions in this booklet.

If you need more room for any answer, use the space provided at the back of this booklet.

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO YOUR TEACHER AT THE END OF THE ALLOTTED TIME.

OVERALL LEVEL OF PERFORMANCE	

You are advised to spend 60 minutes answering the questions in this booklet.

Assessor's use only

QUESTION ONE

(a) At the time of the 2006 Census, 65% of people aged 15 years and over were employed. Of the people employed at the time of the *2006 Census*, 77% were in full-time work and the rest were in part-time work.

Of the people employed full-time:

- 18 % had no qualification
- 33% had a school qualification as their highest qualification
- 49% had a post-school qualification as their highest qualification.

Of the people employed part-time:

- 20% had no qualification
- 43% had a school qualification as their highest qualification
- 37% had a post-school qualification as their highest qualification.

(i) Calculate the proportion of people from the 2006 Census who were employed part-time with no qualification.

$P(Emp \cap PT \cap NoQ)$

$= 0.65 \times 0.23 \times 0.2$

$= 0.0299$

$= 0.03$

(ii) If two people were randomly selected from the 2006 Census, calculate the probability that both were employed full-time. Justify any assumptions that you have made in your calculation of this probability.

$P(Emp \cap FT)$
$= 0.65 \times 0.77$

$= 0.5005$

$P(Both) = 0.5005^2 = 0.251$

Assumption is they are independent this is reasonable given the large number of people involved.

(b) It was found that of a group of 120 people from Wellington:

- 32 are at least 65 years old
- 83 earn at least \$50 000
- 17 are under 65 years old and earn under \$50 000.

(i) Calculate the probability that a randomly selected person from this group earns at least \$50 000 and is under 65 years old. Hence determine with reasoning if the events 'a person from this group earns at least \$50 000' and 'a person from this group is under 65 years old' are mutually exclusive.

	<65	>65	
>50	71	12	83
<50	17	20	37
	88	32	120

$\frac{71}{120}$

= 0.592

As this is not 0 earning at least \$50k and being <65 years old are not mutually exclusive.

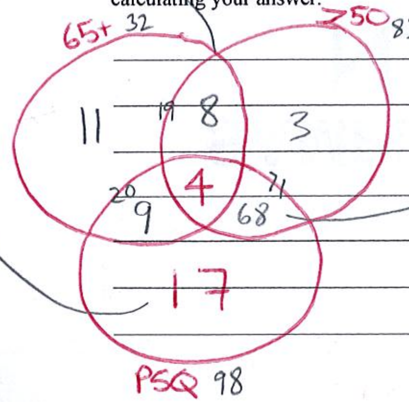
It was also found that of this group:

- 98 have a post-school qualification
- everyone is either at least 65 years old, or earns at least \$50 000, or has a post-school qualification
- 19 are at least 65 years old but do not have a post-school qualification
- 4 are at least 65 years old, earn at least \$50 000, and have a post-school qualification.

(ii) Calculate the probability that a randomly selected person from this group has a post-school qualification and earns at least \$50 000. You should explain your reasoning in calculating your answer.

add to 20

20-4



$\frac{72}{120}$

= 0.6

QUESTION TWO

(a) The data below shows internet access by household type for the West Coast of the South Island.

	Household of a privately owned house	Household of a rented house	
Access to the internet	26.1 %	40.4 %	66.5
No access to the internet	13.7 %	19.8 %	33.5
	39.8	60.2	

(i) Consider the events 'a household of a privately owned house' and 'a household has access to the internet'. Explain whether these events are independent.

$P(Int \cap Pri) = 0.261$

$P(Int) \times P(Pri) = 0.398 \times 0.665$

$= 0.265$

∴ Events are not independent as $P(Int) \times P(Pri) \neq P(Int \cap Pri)$

(ii) Is the household of a privately owned house more likely to have access to the internet than the household of a rented house? Support your answer with appropriate statistical statements.

$P(Int / Pri) = \frac{0.261}{0.398} = 0.656$

$P(Int / Rent) = \frac{0.404}{0.602} = 0.671$

No, the rented house is more likely to have internet.

(b) The Newborn Metabolic Screening Programme screens newborn babies for different metabolic disorders.

Screening attempts to identify babies who have a metabolic disorder. These babies are then referred for diagnostic testing to confirm or rule out having a metabolic disorder.

For a particular metabolic disorder:

- screening identifies about 18 in 20 000 babies as being more likely than others to have this disorder
- subsequent diagnostic testing (including retesting) finds around 10% of babies identified through screening actually have this disorder
- approximately 6 in 66 400 babies actually have this disorder.

If a baby actually has this disorder, what is the approximate risk of not being diagnosed with this disorder through the screening process?

P(screening ∩ sub testing)

$= \frac{18}{20000} \times 0.1 = 0.00009$

P(not id through screening)

$= \frac{6}{66400} - 0.00009$

$= 0.0000003614$ or 3.614×10^{-7}

Risk $= \frac{3.614 \times 10^{-7}}{\frac{6}{66400}}$

$= 0.004$

∴ Risk is 0.4%

Assessor's use only

QUESTION THREE

Emma was given a set of six different keys to her new house. One of the keys opened the dead lock on the front door, and a different key opened the door lock on the front door.

Emma did not know which keys were the correct keys for each lock, and was able to open both locks after four key attempts.

Emma's friend Sene thought this was a low number of key attempts, and wondered what process Emma used to find the correct keys.

Sene designed and carried out a simulation to estimate how many key attempts it would take before both locks were open, using a 'trial and error' process, to see if this might have been the process Emma used.

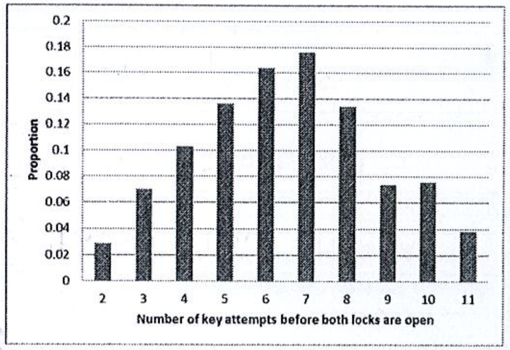
2 locks
6 keys

For the 'trial and error' process, Sene assumed:

- that a key was selected at random to try to open the dead lock
- once Emma had tried a key for the dead lock, she did not try it again
- once Emma found the correct key for the dead lock, she removed this from the set of keys and tried the same process with the door lock.

The results of Sene's simulation are shown below:

Number of key attempts before both locks are open	Frequency
2	29
3	70
4	103
5	136
6	164
7	176
8	134
9	74
10	76
11	38



(a) Calculate the theoretical probability of a person using a 'trial and error' process taking more than two key attempts before both locks are open. Compare this probability with the results from Sene's simulation and discuss any differences.

P(2 key attempts) $= \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$

∴ P(>2 attempts) $= 1 - \frac{1}{30}$

$= \frac{29}{30} = 0.9667$

Assessor's use only

Simulation

$1 - \frac{29}{1000}$

$= 0.971$

Very similar results

Question Three continues on page 8.

Assessor's use only

(b) Sene used the central 90% of her simulation results to check if the number of key attempts Emma took (four) was likely if the 'trial and error' process was used, and concluded it was. Discuss if the results of Sene's simulation support this conclusion.

90% between 3 - 10 attempts.

4 is within this range

∴ simulation results are appropriate.

(c) Calculate the theoretical probability that a person using Emma's process takes four key attempts before both locks are open.

1 0 0 1 $\quad \frac{1}{6} \times \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{30}$

0 1 0 1 $\quad \frac{5}{6} \times \frac{1}{5} \times \frac{4}{5} \times \frac{1}{4} = \frac{1}{30}$

0 0 1 1 $\quad \frac{5}{6} \times \frac{4}{5} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{30}$

$\frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{1}{10}$