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NZQA

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Level 3 Mathematics and Statistics (Statistics), 2018

91585 Apply probability concepts in solving problems

9.30 a.m. Thursday 22 November 2018
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

(a) A 2017 food marketing study from New Zealand examined 70 websites belonging to the most popular food and drink brands. 24 of these websites were targeted at teenagers, while the others were targeted at the general population. 21 of the websites made a positive health claim, and of these websites, eight were targeted at teenagers.

One of the websites is chosen at random.

(i) Explain why the events "a website makes a positive health claim" and "a website is targeted at the general population" are not mutually exclusive.

Support your answer with at least one calculation.

	T	T'	
$+H$	8	13	21
$+H'$	16	33	49
	24	46	70

$P(+H \cap T') = \frac{13}{70} = 0.1857$

$0.1857 \neq 0$ therefore they are not mutually exclusive

(ii) Does the data from this study support the claim that it is more than twice as likely that a website makes a positive health claim if the website is targeted at teenagers rather than the general population?

Support your answer with calculations.

$P(+H/T) = \frac{8}{24} = 0.3\dot{3}$

$P(+H/G) = \frac{13}{46} = 0.283$

$RR = \frac{0.3\dot{3}}{0.283}$

$= 1.18$

$\therefore$ not twice as likely only 1.18 times

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(b) A model has been developed that predicts the brand of toothpaste someone will purchase based on various characteristics of the person, including variables such as age, income, and marital status. To test the model, 100 people were surveyed, and the actual brand of toothpaste purchased by each person was compared to the predicted brand from the model.

The table below shows the results of this test.

Actual brand	Predicted brand			
	A	B	C	
A	18 ✓	3	5	26
B	5	38 ✓	9	52
C	12	2	8 ✓	22
	35		22	

(i) Calculate the percentage of the predictions that were correct (the predicted brand was the same as the actual brand).

$P(\text{correct}) = \frac{18+38+8}{100}$

$= \frac{64}{100} = 0.64 = 64\%$

(ii) Give ONE potential issue with the appropriateness of the model for the different brands, based on the data provided above.

Support your answer with at least two additional calculations to part (i).

Low rates for both A & C

for C $\frac{8}{22} = 0.364$ (3dp)

for A $\frac{18}{35} = 0.514$ (3dp)

(iii) Three of the people from the survey were selected.

Calculate the probability that the model incorrectly predicted the brand of toothpaste for only one of these three people.

State any assumptions made and explain why these need to be made.

Right Right Wrong

$\frac{64}{100} \times \frac{63}{99} \times \frac{36}{98} \times 3 = 0.4488$

Assume people are selected independently

QUESTION TWO

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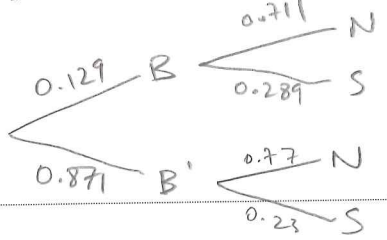
There were 349 secondary schools in New Zealand at the start of 2018. All secondary schools in New Zealand were designated as being in the North Island or South Island.

- 12.9% of these schools are boys' schools
- 71.1% of the boys' schools are in the North Island
- 23.0% of the girls' or co-educational schools are in the South Island

One of these secondary schools is selected at random.

(a) Calculate the probability that the secondary school is in the North Island.



$(0.129 \times 0.711) + (0.871 \times 0.77) = 0.7624$

(b) Let B be the event "a boys' school" and let N be the event "a North Island school".

Calculate and use $P(N \mid B)$ and $P(N)$ to explain whether events B and N are independent.

$P(N/B) = 0.711$

$P(N) = 0.762$

$0.711 \neq 0.762 \therefore$ not independent.

(c) Further analysis of the 349 schools provides the following information:

- All secondary schools in New Zealand were designated as either private or state (which includes partnership schools).
- 16 are private co-educational schools.
- Of the boys' schools, only one is private.
- 331 are state schools.
- 71.0% of state schools are co-educational.

Calculate the probability that a secondary school selected at random is a state girls' school.

0.71 × 331

	B	G	Co-ed	
S	44	52	235	331
P	1	1	16	18
	45	53	251	349.

~~0.129 × 349~~

$\frac{52}{349} = 0.1490$

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Question Two continues on the following page.

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(d) The 349 secondary schools are classified based on the ages of the students on their roll. There are 236 schools classified as Year 9 to 13, and 113 other schools.

A class of Year 13 Statistics students was asked by their teacher to each select a random sample of 50 secondary schools, with replacement, from the 349 secondary schools. The results produced by one student contained 25 schools that were Year 9 to 13 in her sample of 50 secondary schools.

(i) Explain why the teacher might be suspicious that the student did not randomly select the 50 secondary schools.

Support your answer with calculations comparing **expected** and **observed** counts.

Expected Y9-13

$\frac{236}{349} \times 50 = 33.81$

Other

$\frac{113}{349} \times 50 = 16.19$

So unlikely to randomly get 25 'other' schools

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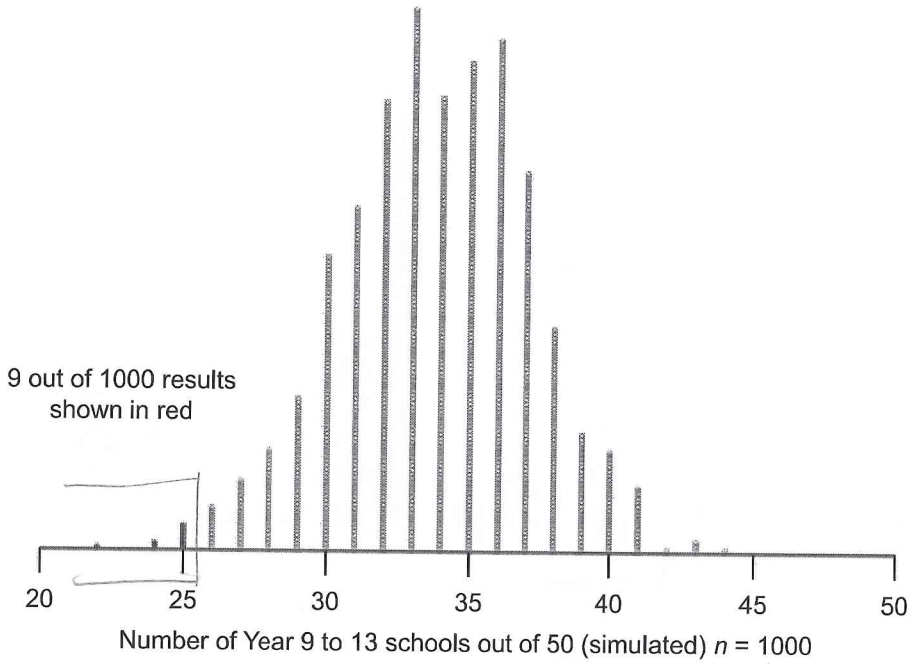
(ii) The teacher designed and carried out a simulation to investigate whether the student's results would have been unlikely, given the student did randomly select the 50 secondary schools. For each trial, she randomly selected 50 schools with replacement from the 349 secondary schools, and recorded how many of these 50 schools were Year 9 to 13 schools.

A summary of the simulation results is shown below (1000 trials).



Based on these simulation results, what conclusion could the teacher make?

Only 9/1000 times were 25 Yr 9-13 schools selected.

Makes students results very unlikely

QUESTION THREE

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United Nations estimates for 2018 based on national censuses from the most recent data from each country were used to construct Graph 1 and Table 1 below.

Graph 1: Percentage of the world's population and the percentage of the world's land area, by continent

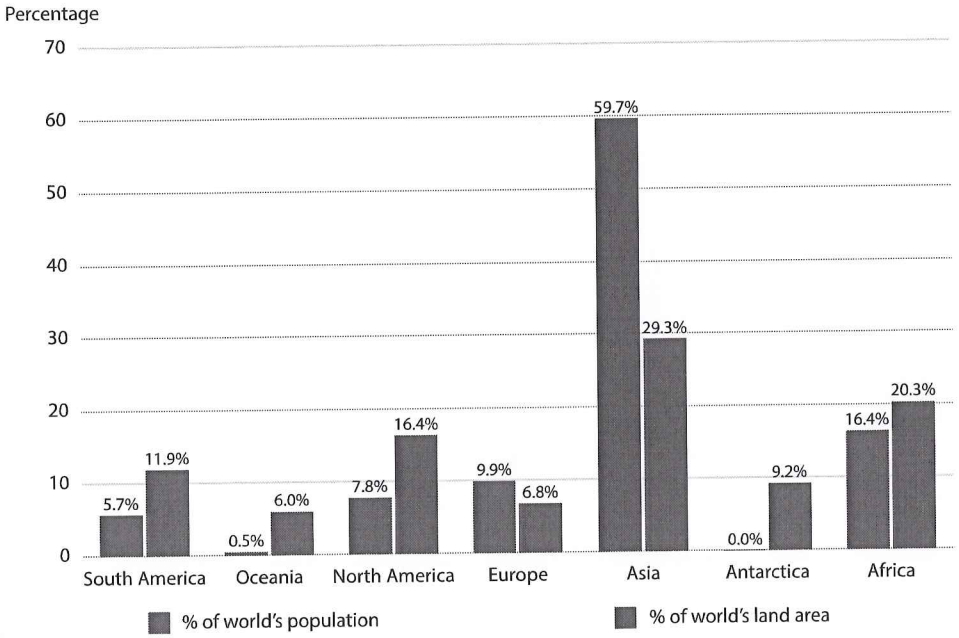

Table 1: Percentage of the population that live in urban areas and the percentage of the population that live in rural areas, by continent

Continent	% of population that live in urban areas	% of population that live in rural areas
South America	80%	20%
Oceania	69%	31%
North America	81%	19%
Europe	74%	26%
Asia	49%	51%
Africa	41%	59%

(a) Use the data provided to calculate an estimate for the percentage of the world's population that live in an urban area of Asia.

$0.597 \times 0.49 = 0.2925$

(b) A student incorrectly reasons that if you know that a person lives in an urban area, then it is likely that this person lives in North or South America.

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(i) Explain how the student has made this incorrect reasoning.

The 80% & 81% are conditional prob and cannot be used to make this assumption.
Most people in SA & NA live in urban locations
not most urban dwellers are in SA & NA

(ii) Based on this data, 54.53% of the world's population live in urban areas.

Calculate an estimate for the probability that a randomly selected person lives in North or South America, given that they live in an urban area.

$$P(A/U) = \frac{P(A \cap U)}{P(U)}$$

$$\frac{(0.057 \times 0.8) + (0.078 \times 0.81)}{0.5453}$$

(SA) (NA)

$$= \frac{0.10878}{0.5453}$$

$$= 0.1995$$

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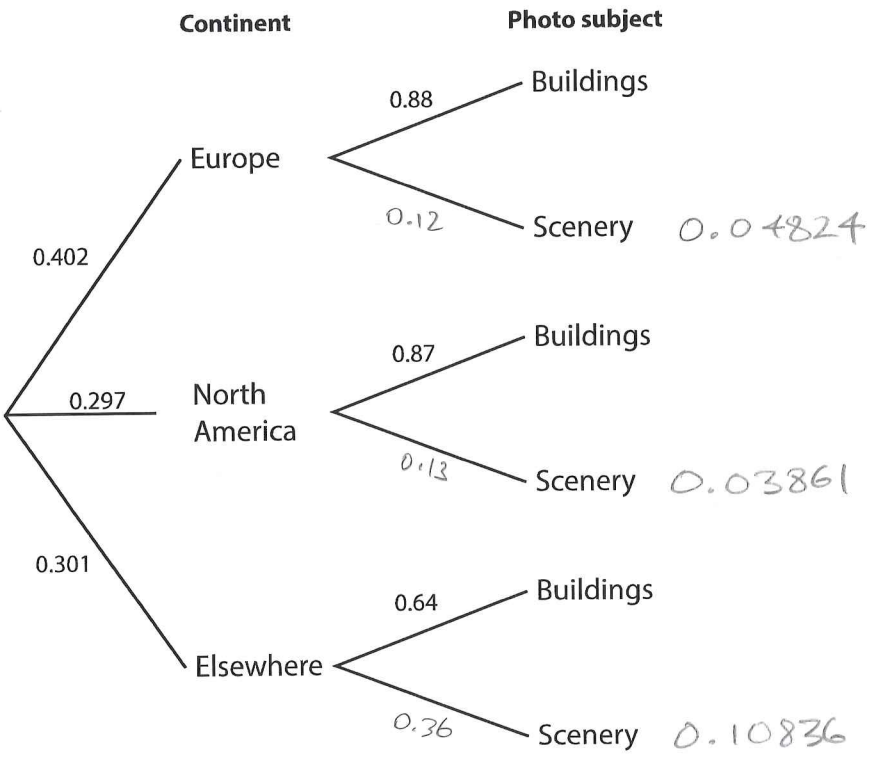
(c) A website displays randomly selected Google Street View photos from around the world. The subjects of these photos can be described as either photos of buildings (including houses) or photos of scenery. Each includes the location of where the photo was taken.

Data from a large number of Google Street View photos displayed by this website were used to develop a probability model. This model has been presented as a partially constructed probability tree below.



(i) Explain how the location of the Google Street View photos displayed by this website and the UN data shown in Graph 1 on page 8, could show that Google Street View does not have photos from all countries in the world.

From graph 1 $16.4 + 6.8 = 23.2$

23.2% of world's land is Eur & NA

$\therefore$ 76.8% is "Elsewhere"

they only photographed 0.301

$0.301 \times 0.768 = 0.23$

Only 30.1% from Elsewhere so unlikely to have pictures from all countries

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(ii) Suppose that a photo displayed on the website is a photo of scenery.

Calculate an estimate for the probability that the location of where the photo was taken is North America.

$$P(NA/SC) = \frac{P(NA \cap SC)}{P(SC)}$$

$$= \frac{0.03861}{0.04824 + 0.03861 + 0.10836}$$

$$= 0.198$$

(iii) A user of the website displays 50 Google Street View photos and finds that 15 of the photos displayed were taken in Europe.

Use this information, and the information presented earlier in part (c) of this question, to discuss the differences between a true probability, a model estimate, and an experimental estimate.

Support your answer with relevant proportions.

True probability is unknown.

The theoretical model probability is 0.402

The experimental is $15/50 = 30\%$ which is lower than the theory.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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