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NZQA

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SUPERVISOR'S USE ONLY

Level 3 Mathematics and Statistics (Statistics), 2019

91585 Apply probability concepts in solving problems

9.30 a.m. Thursday 28 November 2019
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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The school roll data for a provincial New Zealand primary school (Years 0 to 8) by year and gender (as at 1 July 2018) is summarised below:

- There are 343 students.
- Of the 149 female students, 31 are in the Junior School (Years 0, 1, and 2), 65 are in the Middle School (Years 3, 4, and 5), and 53 are in the Senior School (Years 6, 7, and 8).
- Of the male students, 49 are in the Junior School (Years 0, 1, and 2), 73 are in the Middle School (Years 3, 4, and 5), and the remainder are in the Senior School (Years 6, 7, and 8).

Suppose one of these students is chosen at random.

(a) (i) Given that the student is in the Middle School, calculate the probability that they are male.

	J	M	S	Total
F	31	65	53	149
M	49	73	72	194
Total	80	138	125	343

$P(\text{Male}/\text{Middle}) = \frac{73}{138}$

$= 0.529 \quad (3dp)$

(ii) Is it more likely for the student to be male if they are in the Junior School or if they are in the Senior School?

Support your answer with calculations.

$P(\text{Male}/J) = \frac{49}{80} = 0.6125$

$P(\text{Male}/S) = \frac{72}{125} = 0.576$

More likely to be male if in the junior school

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(iii) The principal has used this data to predict that a new student enrolling at the school will be male.

Calculate a proportion using this data that supports the principal's prediction.

$\frac{194}{343} = 0.566$ (3dp)

(iv) Give TWO reasons why care should be taken when using this data to predict that a new student enrolling at the school will be male.

Reason 1: The proportions of male & female may differ in future years.

Reason 2: Popularity of schools change over time, or could have a new school built in the area. Zoning changes.

More on mark schedule

(b) It is generally assumed that 50% of primary school students are male. The principal of the provincial New Zealand primary school referred to in part (a) thinks that, based on their school as a sample of students in their region, the proportion of male students in the region is greater than 50%.

Discuss how carrying out a simulation would help the principal consider whether their students, as a sample of students from the region, could indicate that the proportion of males in their region is in fact greater than 50%.

You do not need to design the simulation.

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Would allow principal to see the variation in the number of males in samples of size 343.

Can then compare with what was observed

QUESTION TWO

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It is suggested that snoring at night has an effect on student performance in examinations. Data was collected on whether or not a group of Year 13 students gained Achievement (or better) in their final examination. It was found that for the 123 snorers, 91 gained Achievement (or better) in their final examination whereas for the 78 non-snorers, 68 gained Achievement (or better) in their final examination.

(a) (i) A student is randomly selected from those in the study.

Let S be the event "student is a snorer" and let E be the event "student gained Achievement (or better) in their final examination".

Compare $P(E \mid S)$ and $P(E)$, and explain what this means for the independence of events S and E.

$P(E/S) = \frac{P(E \cap S)}{P(S)}$

$= \frac{91}{123} = 0.740$

Answers not equal ∴ not independent.

$P(E) = \frac{91+68}{78+123}$

$= \frac{159}{201} = 0.791$

(ii) Estimate how the risk of receiving Not Achieved in their examination compares for Year 13 students who snore and Year 13 students who don't snore.

$P(\text{not Ach} / \text{snorer}) = \frac{32}{123} = 0.2602$

$P(\text{not Ach} / \text{not snorer}) = \frac{10}{78} = 0.1282$

RR 2.03 times as likely to fail if snorer compared with non snorer.

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(iii) For students who snore, 32% complete all of their homework tasks. For students who don't snore, 65% complete all of their homework tasks.

Suppose three students are chosen at random from a large group of Year 13 students.

Calculate the probability that all three Year 13 students don't snore and always complete all of their homework tasks.

Support your answer with statistical statements and reasoning, including any assumption(s) made.

$\frac{78}{201} \times 0.65 = \frac{169}{670}$ or 0.2522

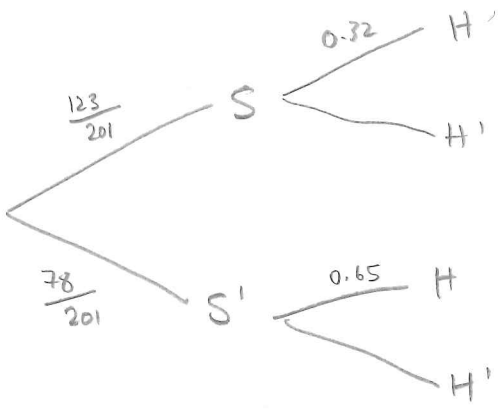
$P(\text{all } 3) = 0.2522 \times 0.2522 \times 0.2522$

or 0.2522^3

$= 0.016$

Assumptions

- Snoring & homework habits are independent
- Prob of snoring is the same as prior question
- Assume sample large enough that sampling without replacement and prob constant.



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(b) (i) A group of 260 Year 11 students from the same school is sitting an examination. 127 of these Year 11 students self-classify as 'snorers'.

Assuming these 127 students have the same risk of receiving Not Achieved in their examination as the Year 13 students who are also snorers, and that the risk of receiving Not Achieved is the same for non-snorers in Year 11 and non-snorers in Year 13, how many of the group of 260 Year 11 students would you expect to gain Achievement (or better)?

Risk snorer achieving $= \frac{91}{123}$

Risk snorer' achieving $= \frac{68}{78}$

Yr 11 $= \left(\frac{91}{123} \times 127\right) + \left(\frac{68}{78} \times 133\right)$

$= 94 + 116$

$= 210$ students

(ii) Make TWO comments on the validity of the assumption in (b)(i) that the Year 11 students who snore have the same risk of receiving Not Achieved in their examination as the Year 13 students who also snore.

Comment One: Yr 11 may not have the same chance of achieving as Yr 13 students.

Comment Two: Yr 11 student self classified as snorers, not clear how the year 13's were identified.

More reasons in marking schedule.

QUESTION THREE

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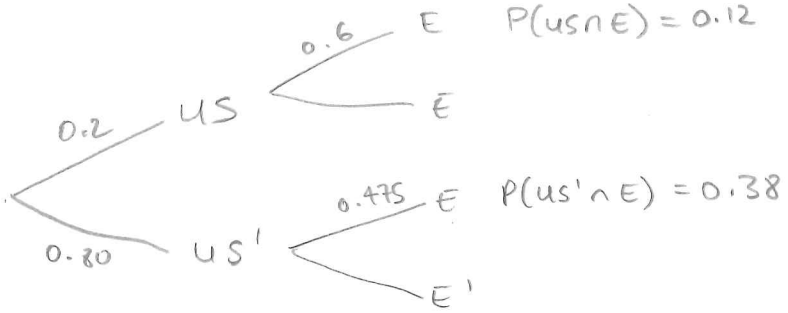
Three of the social media sites commonly used by young adults are Instagram, Snapchat, and Facebook.

Instagram (a photo and video sharing app) reports that around 80% of its users are from outside the United States of America (non-US), with a large proportion of its users logging in daily. Of the non-US users, it is estimated that 47.5% log in every day. Of the US users, 60% log in every day.

(a) (i) There are estimated to be 1 billion Instagram users in the world.

Instagram claims that it has 0.8 billion users logging in every day.

Is this claim consistent with the information provided?



$0.12 + 0.38 = 0.5$

0.5×1 billion $= \frac{1}{2}$ a billion

So the claim is not supported

(ii) Given that a user logs in every day, are they more likely to be a US user or non-US user?

$P(US'/E) = \frac{0.38}{0.5} = 0.76$

$P(US/E) = \frac{0.12}{0.5} = 0.24$

Non US users are more likely to log in everyday.

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(iii) A probability model is developed based on the information provided. Under this model, the probability that an Instagram user from the US logs in daily is 60%.

Discuss why it might not be appropriate to apply this model to ALL TYPES of US Instagram users.

- Not everyone has access to internet everyday
- Some US users will login everday (ie social influences)

(b) If a young adult uses one social media site, they are likely to use the other sites as well.

For a group of 300 young adults:

- 84 use all three of these social media sites.
- 12 use none of these social media sites.
- 207 use Snapchat.
- 195 of the Snapchat users also use Instagram.
- 94 of the Snapchat users also use Facebook.
- 216 use Instagram.
- 163 use Facebook.

(i) Calculate the proportion of these young adults that use both Snapchat and Instagram, but not Facebook.

		I	I'	
F	S	84	10	163
	S'	9	60	
F'	S	111	2	137
	S'	12	12	
		216	84	300

195 − 84

207 − 84 − 10 − 111

$$\frac{111}{300} = 0.37$$

Question Three continues on the following page.

(ii) It is claimed that if a social media user randomly selected from those surveyed is an Instagram user, they are more than twice as likely to also be a Snapchat user than a Facebook user.

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Is this claim correct?

Support your answer with appropriate statistical statements.

$$P(S/I) = \frac{195}{216} = 0.9028$$

$$P(F/I) = \frac{93}{216} = 0.4306$$

$$RR = \frac{0.9028}{0.4306}$$

$$= 2.097$$

∴ the claim is correct they are twice as likely to be a snapchat user than a facebook user given they also use intagram.