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NZQA

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Tick this box if there is no writing in this booklet

Level 3 Mathematics and Statistics (Statistics) 2020

91585 Apply probability concepts in solving problems

9.30 a.m. Wednesday 18 November 2020
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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After exercise, the blood sugar and dehydration levels of 80 Year 13 students from one school were measured. Blood sugar levels were classified as 'low blood sugar' or 'normal blood sugar' and dehydration levels were classified as 'dehydrated' or 'not dehydrated'.

Of the 32 students with low blood sugar levels, 20 were also dehydrated.

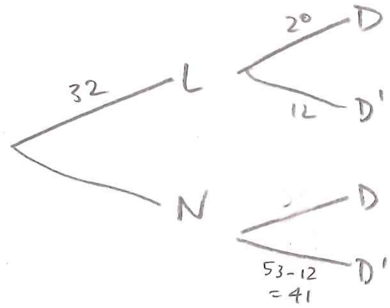
53 students were not dehydrated.

(a) One of the students was randomly selected.

(i) Calculate the probability that the student was dehydrated.

	D	D'	
Low	20	12	32
Normal	7	41	48
	27	53	80

or



$P(\text{dehydrated}) = \frac{27}{80}$

$= 0.3375$

(ii) Explain whether the events 'student is dehydrated' and 'student has low blood sugar' are mutually exclusive.

Support your answer with statistical reasoning.

~~Be~~ $P(D \cap \text{Low}) = \frac{20}{80}$

$= 0.25$

Both dehydrated and low blood sugar can happen so these events are not mutually exclusive.

The probability would have to be zero for them to be mutually exclusive.

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(iii) Give TWO reasons why care should be taken when using this data to estimate the probability that any randomly chosen Year 13 student in New Zealand shows low blood sugar levels after exercise.

Reason One: Students only from one school might be different in other schools

Reason Two: Only 80 students which is not a big sample.

(b) Both dehydration and low blood sugar levels are thought to decrease cognitive (thinking) ability.

In a separate study involving a different group of Year 13 students from the same school, after exercise it was observed that:

- for the 15% of students who were dehydrated and who had low blood sugar levels, 45% showed decreased cognitive ability
- for the 57% of students who were not dehydrated and had normal blood sugar levels, 5% showed decreased cognitive ability
- for all other students, 32% showed decreased cognitive ability.

(i) Calculate the probability that a randomly selected student in this group showed decreased cognitive ability after exercise.

Deh $0.15 \times 0.45 = 0.0675$

Not Deh $0.57 \times 0.05 = 0.0285$

Other $0.32 \times (1 - 0.57 - 0.15)$

$0.32 \times 0.28 = 0.0896$

'other' students

$0.0896 + 0.0285 + 0.0675 = 0.1856$

denominator out of these.

(ii) Calculate the proportion of students in this study with decreased cognitive ability that are not dehydrated and have normal blood sugar levels.

Interpret this proportion.

$$\frac{P(\text{Decreased and not deh})}{P(\text{decreased})}$$

$$= \frac{0.0285}{0.1856}$$

$$= 0.1536$$

Around 15% of those with decreased cognitive ability are not dehydrated and have normal blood sugar levels.

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QUESTION TWO

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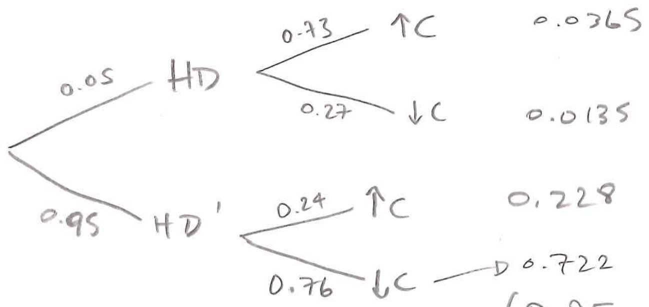
As part of enrolling at a medical clinic, new patients undergo health-screening tests. One of these screening tests measures the amount of blood cholesterol in order to identify those patients who may have heart disease. Cholesterol levels greater than 200 mg/dL suggest that the patient may have heart disease.

Medical studies report that:

- approximately 5% of the New Zealand population are known to have heart disease
- 73% of people with heart disease have cholesterol levels greater than 200 mg/dL, and
- 76% of people without heart disease have cholesterol levels of 200 mg/dL or less.

(a) (i) Out of 100 individuals who are screened with this test, approximately how many would be expected to have a cholesterol level greater than 200 mg/dL?



$(0.05 \times 0.73) + (0.95 \times 0.24) \times 100$

$= 26.45$ so 26 or 27 people

(ii) A patient is told that the result of their screening test is positive (cholesterol level is greater than 200 mg/dL).

Comment on whether this patient should be concerned that they do actually have heart disease.

Support your answer with statistical reasoning.

$$P(HD/Positive) = \frac{P(HD \cap Positive)}{P(Pos)}$$

$$= \frac{0.0365}{(0.0365 + 0.228)}$$

$$= \frac{0.0365}{0.2645} = 0.1380$$

Should not be concerned as only 13-14% chance.

(iii) Suppose the threshold value for suggesting a person has heart disease is changed from 200 mg/dL to 250 mg/dL.

Describe how this increase in threshold value could change the proportion of patients correctly identified as having heart disease (those with a cholesterol level greater than the threshold, that actually have heart disease).

If threshold increases, P(positive) decreases

See mark schedule for elaboration.

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(b) The medical clinic is interested in understanding the probability of patients being diagnosed with heart disease, diabetes, and stroke.

After analysing the records of 5000 patients, they found that:

- 71 were diagnosed with heart disease, diabetes, and stroke
- 1359 were not diagnosed with any of these conditions
- 1907 were diagnosed with heart disease
- 1814 were diagnosed with diabetes
- 627 were diagnosed with stroke
- 388 of those diagnosed with heart disease were also diagnosed with diabetes
- 170 of those diagnosed with heart disease were also diagnosed with stroke.

(i) Calculate the proportion of patients that were diagnosed with heart disease, but not diabetes or stroke.

		S		S'	
HD	D	71 ~~170~~	388	317 ~~1737~~	1907
	D'	99		1420	
HD'	D	149	1426	1277	3093
	D'	308	1667	1359	
		627		4373	5000

$\frac{1420}{5000} = 0.284$

(ii) It is claimed that, for patients with heart disease, they are twice as likely to be diagnosed with diabetes than stroke.

Does the data support this claim?

Support your answer with appropriate statistical statements.

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$$\frac{D \cap HD}{HD} = \frac{388}{1907} = 0.2035$$

$$\frac{S \cap HD}{HD} = \frac{170}{1907} = 0.0891$$

$$RR = \frac{0.2035}{0.0891}$$

$$= 2.282$$

Claim is justified, over twice as likely

QUESTION THREE

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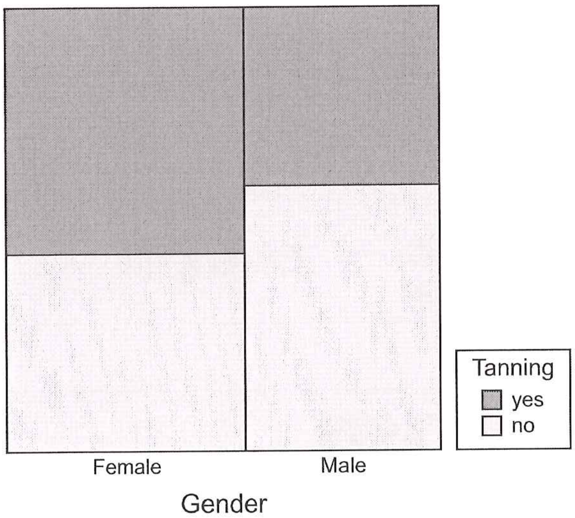
Research suggests that artificial tanning is detrimental to health. The Eikosogram below shows a representation of the proportions of a group of male and female students who have participated in artificial tanning during the previous year. The rectangular regions of the Eikosogram have areas in proportion to the probabilities of participating in tanning (or not) for males and females in a study group.



There were 250 students in this study, of whom 55.2% were female. Just less than half (48.4%) of the 250 students had participated in artificial tanning during the previous year (including $\frac{2}{5}$ of the males).

(a) (i) Represent the information provided about the study using a two-way table of counts.

2/5 of 112 rounded up

0.484×250

	F	M	
T	76	45	121
T'	62	67	129
	138	112	250

250×0.552

(ii) Explain what can be learned from this data about the potential relationship between gender and tanning for participants in this study.

Support your answer with calculations.

$$P(\text{tan} / \text{fem}) = \frac{\text{tan} \,\&\, \text{fem}}{\text{fem}}$$

$$= \frac{76}{138} = 0.5507$$

$$P(\text{tan} / \text{male}) = \frac{\text{tan} \,\&\, \text{male}}{\text{male}}$$

$$= \frac{45}{112} = 0.4018$$

Females more likely than male.

(iii) It is claimed that females are 1.5 times as likely to participate in artificial tanning, compared to males.

Does this data support this claim?

Support your answer with appropriate statistical statements, which may include reasoning based on the Eikosogram.

$$RR = \frac{0.5507}{0.4018}$$

$$= 1.37$$

Females 1.37 as likely to tan which is not 1.5.

Data does not support the claim

Question Three continues on the following page.

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(b) The counts of these 250 students who have ear piercing(s) is summarised in the two-way table below. A student is randomly chosen from this group.

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	Ear piercing(s)	No ear piercing(s)	
Male	58	54	112
Female	91	47	138
	149	101	250

(i) Explain whether the events 'student is female' and 'student has ear piercing(s)' are independent.

Support your answer with statistical statements and reasoning.

$P(\text{female} \cap \text{pierced}) = \frac{91}{250} = 0.364$

$P(\text{female}) \times P(\text{pierced}) = \frac{138}{250} \times \frac{149}{250}$

$= 0.328992$

$0.364 \neq 0.328992 \therefore$ not independent

(ii) Three male students are randomly chosen from this group.

Calculate the probability that two or more of these three male students have ear piercing(s).

Support your answer with statistical statements and reasoning.

Include a discussion of the effect of any assumption(s) made.

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$P(\text{all } 3) = \frac{58}{112} \times \frac{57}{111} \times \frac{56}{110} = 0.1354$

$P(\text{just } 2) = \left(\frac{58}{112} \times \frac{57}{111} \times \frac{54}{110}\right)3 \leftarrow$

$= 0.3916$

PP NP
NP P P
P NP P

3 diff orders,

$0.1354 + 0.3916 = 0.5270$

Assumptions

- Independence between piercings
- Assumed sampling without replacement.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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91585