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NZQA

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Level 3 Mathematics and Statistics (Statistics), 2017

91585 Apply probability concepts in solving problems

9.30 a.m. Monday 27 November 2017
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–8 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

A sample of 996 students in Years 9 to 13 was taken from the Census at School 2015 database.

(a) 78.4% of these students were born in New Zealand.

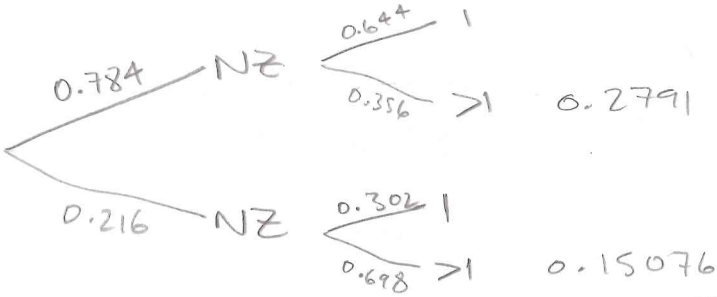
100% of these students can speak at least one language fluently.

Of the students born in New Zealand, 35.6% can speak more than one language fluently.

Of the students not born in New Zealand, 69.8% can speak more than one language fluently.

A student from the sample is chosen at random.

(i) Calculate the probability that the student can speak only one language fluently.



$(0.784 \times 0.644) + (0.216 \times 0.302) = 0.5701$

(ii) Explain why the events "a student was born in New Zealand" and "a student speaks more than one language fluently" are not independent.

$P(NZ \cap >1) = 0.784 \times 0.356$

$= 0.2791$

$P(NZ) \times P(>1) = 0.784 \times (0.2791 \times 0.150768)$

$= 0.3370$

$0.3370 \neq 0.2791$

Therefore not equal not independent

(b) The following tables were created using further data from the 996 students.

Gender	Owns a cell phone	
	Yes	No
Female	481	52
Male	408	55

Owns a cell phone	Has a Facebook account	
	Yes	No
Yes	750	139
No	64	43

Gender	Has a Facebook account	
	Yes	No
Female	433	100
Male	381	82

A student from the sample is chosen at random.

(i) Calculate the probability that the student is female and does not own a cell phone.

$\frac{52}{996} = 0.0522$

(ii) Are the events "has a Facebook account" and "owns a cell phone" mutually exclusive? Support your answer with appropriate statistical statements.

$P(F \cap C) = \frac{750}{996}$

$\frac{750}{996} \neq 0$ ∴ not mutually exclusive

(iii) 349 students in this sample were male, had a Facebook account, and owned a cell phone.

Calculate the probability that the student is female, does not have a Facebook account, and does not own a cell phone.

		M	F	
F	C	349	750 − 349 = 401	814
	C′	32	433 − 401 = 32	
F′	C	408 − 349 = 59	481 − 401 = 80	182
	C′	23	20	
		463	533	996

$\frac{20}{996}$

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QUESTION TWO

(a) Data was obtained on all flights that departed from Wellington Airport during one day in January 2017.

For the 83 flights that had departure time data available:

- 64 flights were operated by Air New Zealand
- 31 flights were delayed
- 12 flights were not operated by Air New Zealand and were not delayed.

(i) Suppose one of these flights is chosen at random.

Calculate the probability that this flight was delayed, given that the flight was not operated by Air New Zealand.

	D	D'	
ANZ	24	40	64
ANZ'	7	12	19
	31	52	83

$$P(D/ANZ') = \frac{P(D \cap ANZ')}{P(ANZ')} = \frac{7}{19} = 0.368$$

(ii) Give TWO reasons why care should be taken when using this data to predict whether the next flight departing from Wellington Airport will be delayed.

1. Limited sample size

2. Next day - diff conditions

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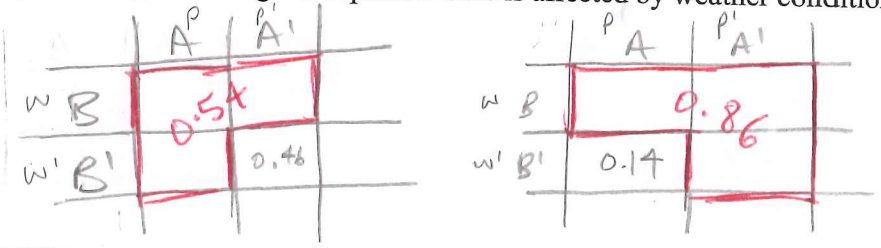
(iii) A probability model has been developed for flights departing from another airport.

Let A be the event "a flight's departure time is affected by passenger behaviour".

Let B be the event "a flight's departure time is affected by weather conditions".

Under this model, $P(A \cup B) = 0.54$ and $P(A' \cup B) = 0.86$.

What is the probability that a flight's departure time is affected by weather conditions?



$P(B') = 0.46 + 0.14 = 0.6$

$\therefore P(B) = 1 - 0.6 = 0.4$

(b) A website has developed a model that predicts a person's gender based on a sample of that person's formal writing. After predicting each person's gender, the website asks each person to select their gender (female or male). The table below shows the results for a random sample of 400 people who used the website to predict their gender.

Selected gender	Predicted gender		
	Female	Male	
Female	172 ✓	26	198
Male	108	94 ✓	202
	280	120	

(i) Calculate the percentage of the predictions that were correct (the predicted gender was the same as the selected gender).

$\frac{172 + 94}{400} = 0.665$

$\therefore 66.5\%$

(ii) Give ONE potential issue with the appropriateness of the model used by the website, based on the data provided above.

Support your answer with at least one calculation.

Predicts females better than males

♀ $\frac{172}{198} = 0.869$ (3dp)

♂ $\frac{94}{202} = 0.465$ (3dp)

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QUESTION THREE

(a) Strep throat is an infection of the back of the throat and the tonsils. Rapid antigen detection tests (RADTs) give either a positive or negative result for strep throat, but are not 100% accurate. A study was conducted with 298 primary school children who had sore throats. After the RADT was used, another test was used to confirm whether each child had strep throat or not. Data from this study is shown in the table below.

	Had strep throat	Did not have strep throat	
Positive RADT result	0.074	0.124	0.198
Negative RADT result	0.131	0.671	0.802
	0.205	0.795	

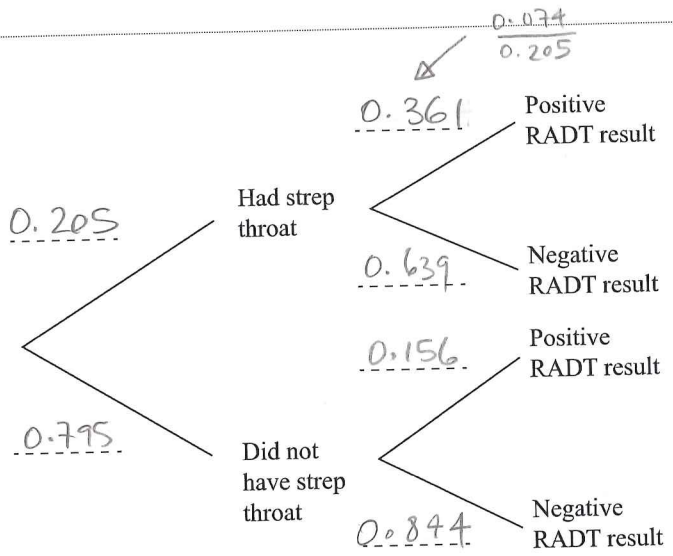
(i) A website offering health advice for New Zealand parents states that "… most sore throats for children are not strep throat".

Does the data from this study support this statement?

P(not strep) = 0.795

this supports the statement as over 50%

(ii) Use the information provided to calculate the necessary probabilities to complete the probability tree shown below, rounding probabilities to 3 decimal places.



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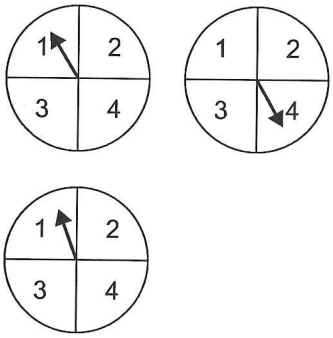
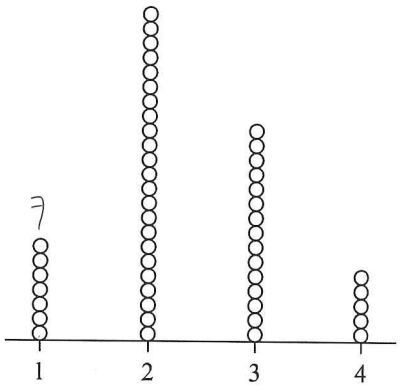
(b) A game involves spinning three spinners. The score for the game is the median of the three numbers that the spinners land on. A person has played this game 50 times. The score for each game is shown on the dot plot below.



Example of one game (score = 1)



Scores from 50 games

(i) Calculate an estimate for the probability of gaining a score of one, using the data in the dot plot.

$7/50 = 0.14$

(ii) Calculate the theoretical probability of gaining a score of one, assuming each spinner is equally likely to land on each of the four numbers shown.

Support your answer with appropriate statistical statements or diagrams.

Outcomes:

111 121 211
112 131 311
113 141 411
114

$\frac{10}{64} = 0.156$ (3dp)

Score has to be 1, 1, x

$4 \times 4 \times 4 = 64$ possible outcomes.

(iii) Complete the theoretical probability distribution table for S, the score for the game.

s	1	2 $^{22}/_{64}$	3 $^{22}/_{64}$	4 $^{10}/_{64}$
$P(S = s)$	0.156	0.344	0.344	0.156

as above $^{10}/_{64}$

Same as 1 but 4.

Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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