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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Level 2 Mathematics and Statistics, 2013

91267 Apply probability methods in solving problems

2.00 pm Monday 18 November 2013
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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The examination starts on the following page.

You are advised to spend 60 minutes answering the questions in this booklet.

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QUESTION ONE

A team from school A and a team from school B are in the final of a school sport competition.

The two teams take part in a best-of-three game series.

Each game continues until one team has a winning score.

The team that wins two games wins the competition.

(a) The teams are evenly matched.

(i) Find the probability that team A will win the competition at the end of the first two games.

$P(AA) = 0.5 \times 0.5$

$= 0.25$ or $1/4$

(ii) What is the probability that the winner cannot be decided until three games have been played?

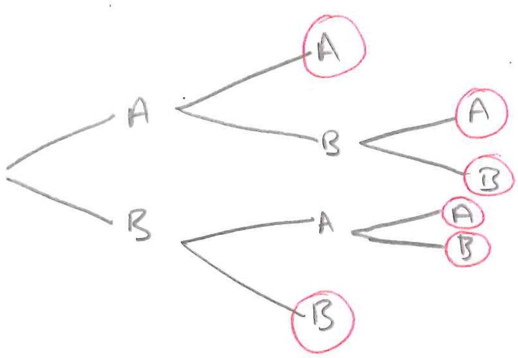
$P(3 \text{ games}) = 0.5 \times 0.5 \times 0.5$

$= 0.125$

4 ways this can happen

$= 0.125 \times 4$

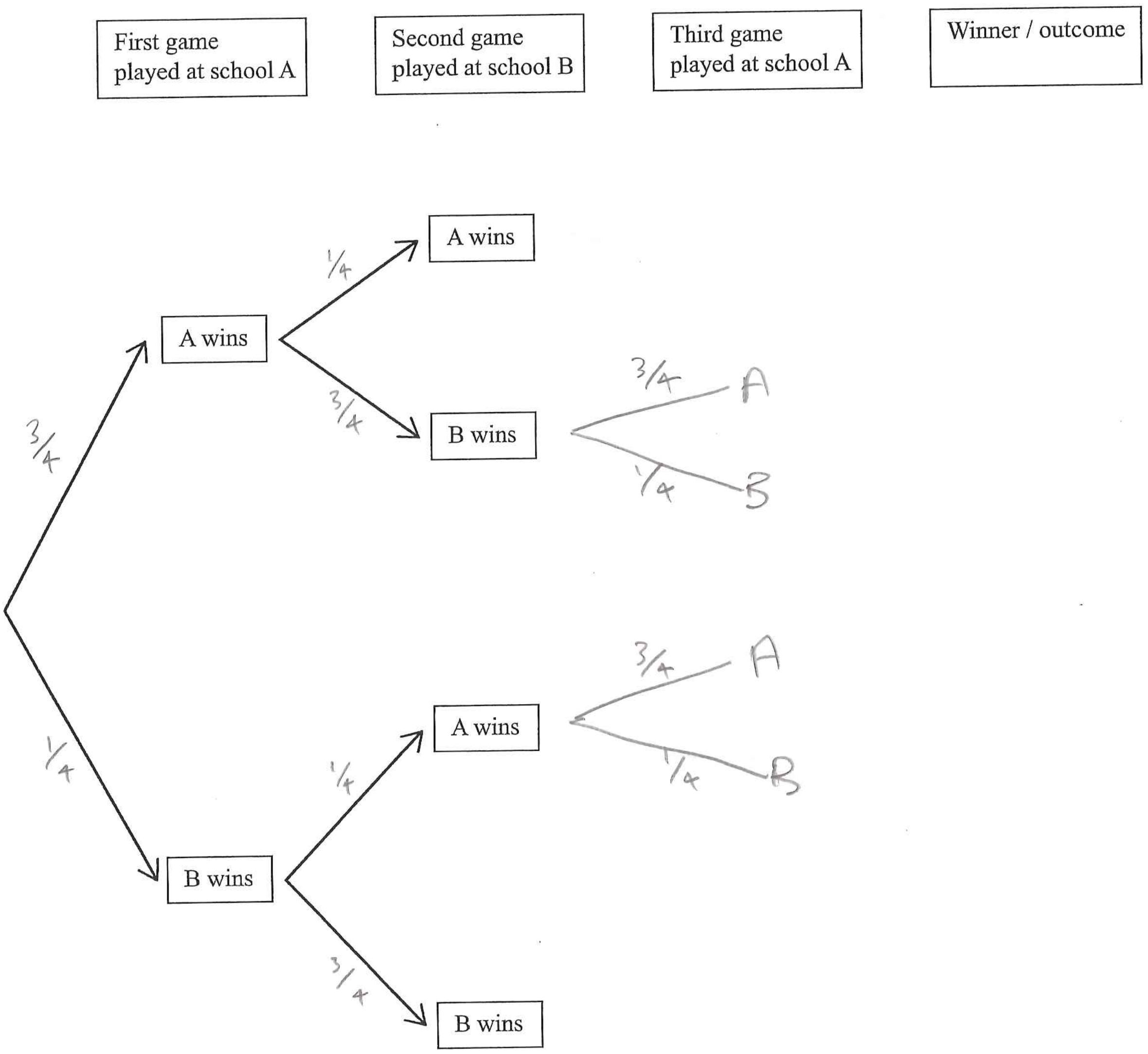
$= 0.5$



(b) Bhavik wants to see if the team that has the game at their own school twice has an advantage.

The first game is played at school A, the second at school B, and the third at school A.

From the records, he finds that the probability of either team winning at their own school is ¾. He begins to make a tree diagram as shown below.



(i) What is the probability that school A wins the competition?

$P(AA) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

$P(ABA) = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$

$P(BAA) = \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{64}$

$\frac{3}{16} + \frac{27}{64} + \frac{3}{64} = \frac{21}{32}$

$= 0.84375$

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(ii) If team A wins the competition, what is the probability that the winner would not have been decided until the end of the third game?

$$\frac{P(\text{A wins in } 3)}{P(\text{A wins})} = \frac{\frac{30}{64}}{\frac{21}{32}} = \frac{0.46875}{0.65625}$$

$= 0.71429$

(iii) Bhavik said that team A is almost twice as likely to win the competition as team B.

Is Bhavik's claim justified?

Show your reasoning clearly.

$P(\text{B win}) = 1 - (\text{A winning})$

$= 1 - \frac{21}{32}$

$= \frac{11}{32}$

$RR = \frac{\frac{21}{32}}{\frac{11}{32}}$

$= 1.9$

Almost true but not quite twice as likely

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(c) **Three games** need to be played to decide the winner of the competition.

The first game is played at school A, the second at school B, and the third at school A.

The probability of either team winning a game played at their school is p.

Show that the probability team A will win the competition compared to the probability team B will win is given by $\frac{p}{1-p}$.

A on top for RR

ABA or BAA

$P(\text{A wins}) = p^3 + (1-p)\times(1-p)\times p$

$= p^3 + p(1-p)^2$

BAB ABB

$P(\text{B wins}) = (1-p)\times(1-p)(1-p) + p\times p\times(1-p)$

$= (1-p)^3 + p^2(1-p)$

$$RR = \frac{p^3 + p(1-p)^2}{(1-p)^3 + p^2(1-p)}$$

factorise $= \frac{p\cancel{(p^2+(1-p)^2)}}{1-p\cancel{(1-p^2+p^2)}} = \frac{p}{1-p}$

QUESTION TWO

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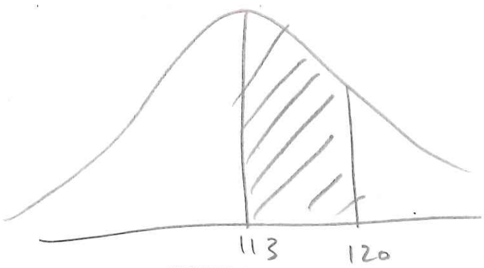
Systolic blood pressure measures the pressure of blood in the arteries as the heart beats and is measured in mm Hg (millimetres of mercury).

In this question "blood pressure" refers to "systolic blood pressure".

The blood pressure of all the students in a school where Alice is the nurse, is approximately normally distributed, with mean 113 mm Hg, and standard deviation 10.3 mm Hg.

(a) (i) What proportion of the students, chosen at random from Alice's school, would be expected to have blood pressure between 113 mm Hg and 120 mm Hg?



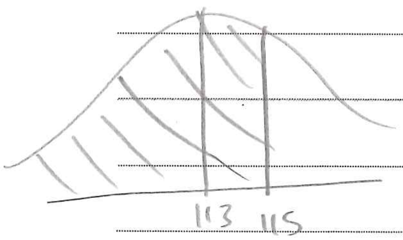
$z = \dfrac{120 - 113}{10.3}$

$= 0.680$

$= 0.2518$

(ii) There are 719 students at Alice's school.

How many students would be expected to have blood pressure less than 115 mm Hg?



$z = \dfrac{115 - 113}{10.3}$

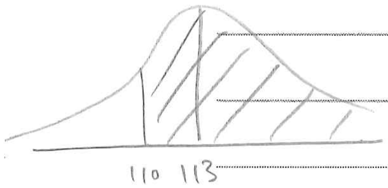
$= 0.194$

$0.077 + 0.5 = 0.577$

$0.577 \times 719 = 414.863$

$\therefore$ 414 or 415 students.

(iii) What is the probability that two randomly selected blood pressure readings from students at Alice's school are both over 110 mm Hg?



$$z = \frac{110-113}{10.3}$$

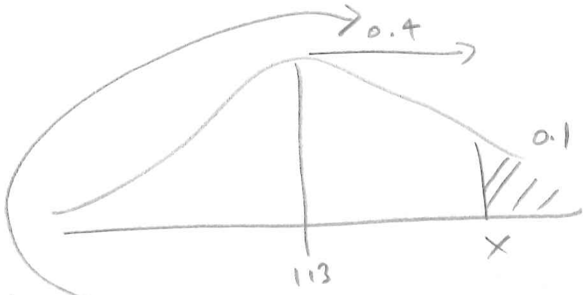
$$= -0.291$$

$$= 0.1145 + 0.5 \qquad 0.6145^2$$

$$= 0.6145 \qquad = 0.3776$$

(iv) Alice decides to retest the 10% of her students who have the highest blood pressures on the first day of term.

What is the lowest blood pressure value of a student that Alice will need to retest?



look up backwards

$$z = \frac{X-113}{10.3}$$

$$1.281 = \frac{X-113}{10.3}$$

$$X = 126.2$$

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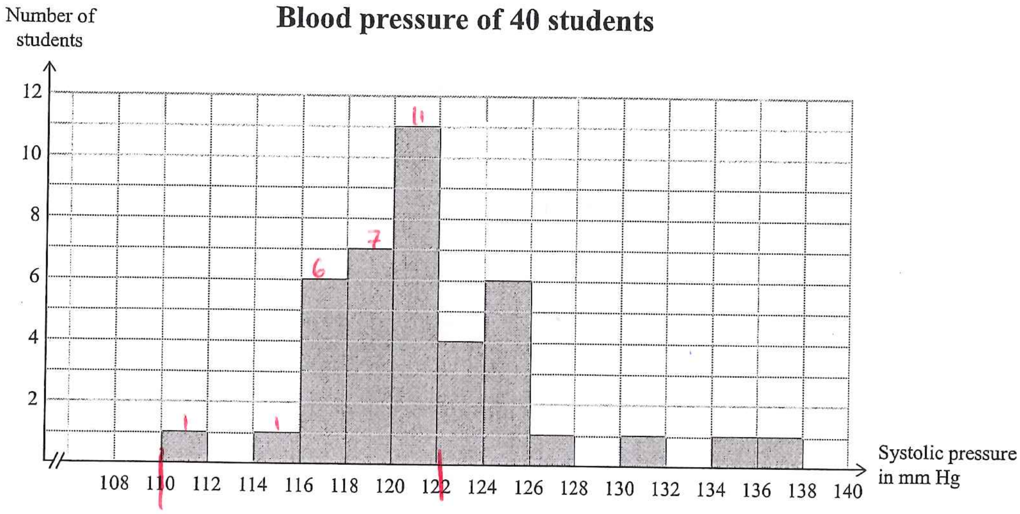
(b) Immediately after Physical Education one day, Alice takes blood pressure readings of 40 students. The results are shown in the following histogram.



(i) What proportion of the blood pressure readings of these students lies between 110 and 122 mm Hg?

$\frac{26}{40}$

(ii) Compare the results for these students with the distribution of results for all the students at Alice's school.

See marking scheme

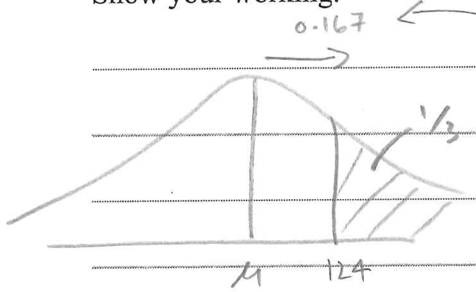
(c) Studies show that the blood pressure of New Zealand Year 13 students tends to be higher than Year 9 students, but remains normally distributed.

Alice checks the blood pressure of her Year 13 students before they leave school.

She finds the standard deviation of this group's blood pressure is now 9.8 mm Hg, and one third of the students have blood pressure above 124 mm Hg.

Find the mean blood pressure of this group.

Show your working. ← look up backwards.



$$z = \frac{124 - \mu}{9.8}$$

$$0.431 = \frac{124 - \mu}{9.8}$$

$\mu = 119.78$ mm Hg

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The examination continues on the following page.

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QUESTION THREE

(a) Blood pressure in this question refers to systolic blood pressure.

High blood pressure can indicate heart disease.

One way to test for heart disease is by measuring a person's blood pressure and seeing if it is high, ie at least 140 mm Hg.

Tests showed that 8 825 of 10 300 people sampled had blood pressure under 140 mm Hg.

Of these, 8 625 did not have heart disease.

1 339 of the people sampled had heart disease.

The information above is summarised in the following table.

	Low blood pressure	**High blood pressure**	**Total**
Did not have heart disease	8 625	336	8 961
Have heart disease	200	1 139	1 339
Total	8 825	1 475	10 300

For the people who were tested:

(i) What proportion of the people had a blood pressure reading that was low?

$\frac{8825}{10300} = 0.8568$

(ii) What proportion of people had a high blood pressure reading or heart disease ?

$1475 + 1339 - 1139 = 1675$

$\frac{1675}{10300} = 0.16$

High blood pressure is used as an indicator of heart disease.

only out of these people

(iii) What percentage of people in the sample with heart disease actually had a high blood pressure reading?

$$\frac{1139}{1339} = 0.8506$$

$\therefore$ 85%

(iv) Based on this sample, what is the risk of a person with high blood pressure having heart disease?

just those with ↑ bp

$$\frac{1139}{1475} = 0.7722$$

(b) Blood pressure in this question refers to systolic blood pressure.

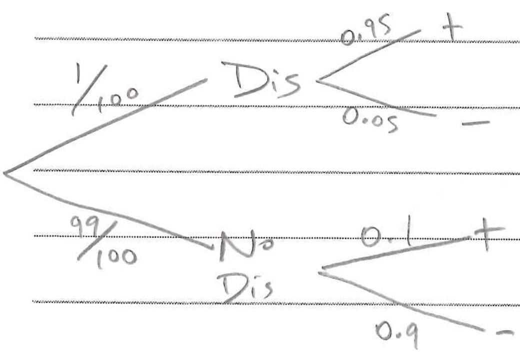
A form of heart disease affects about one in 100 people.

A blood pressure test for this disease gives a correct positive result 95% of the time and a correct negative result 90% of the time. That is, if a person has this disease, then the test says the person has the disease with probability 0.95. If a person does not have the disease, then the test says the person does not with probability 0.90.

A randomly selected person has a positive result for the blood pressure test.

What is the risk of the person actually having the disease?

P(diseased & test positive) or P(no dis but test +)



$0.01 \times 0.95 = 0.0095$

$0.99 \times 0.1 = 0.099$

$0.0095 + 0.099 = 0.1085$

$$\frac{0.0095}{0.1085} = 0.0876$$

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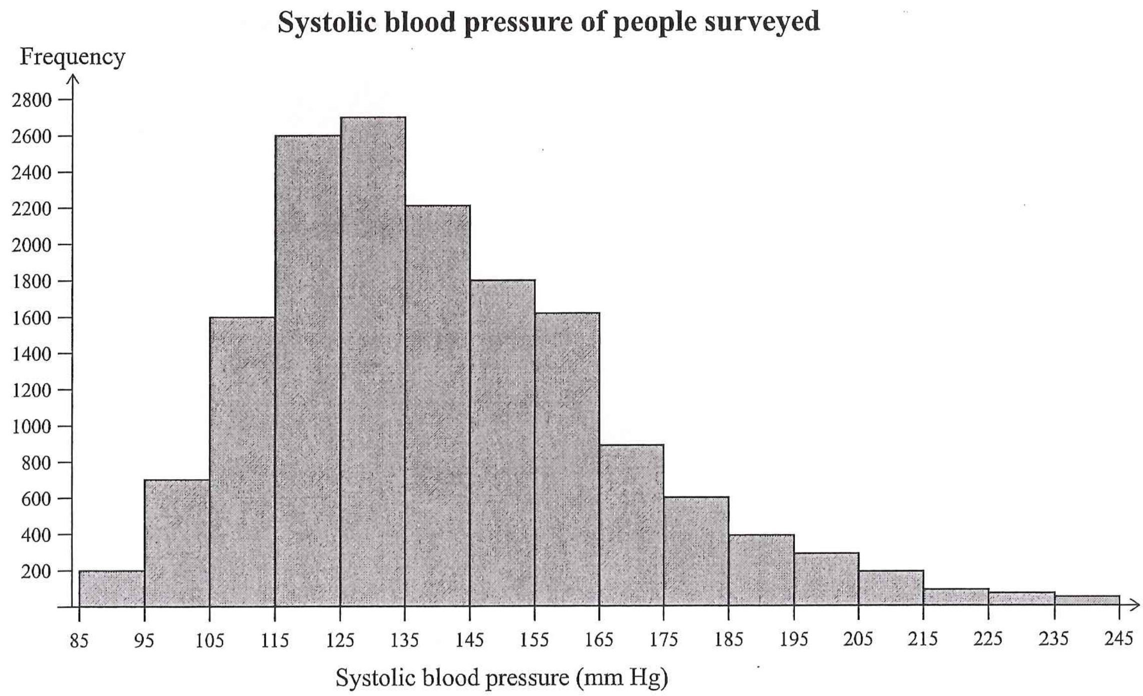
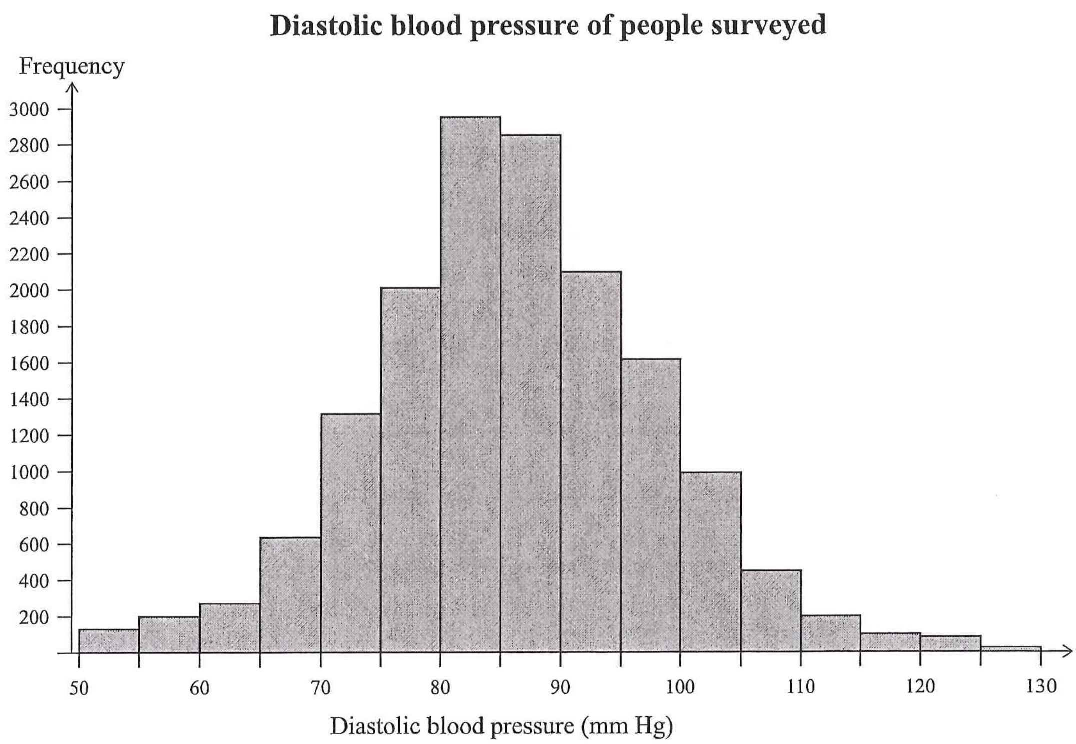
(c) Systolic blood pressure is the pressure as the heart beats, and diastolic blood pressure is the pressure as the heart muscles relax. Both readings are important when blood pressure is taken.

The graphs below show the systolic and diastolic blood pressure in a population taken from a survey of over 16 000 people.





Figures adapted from *The Blood Pressure "Uncertainty Range" – a pragmatic approach to overcome current diagnostic uncertainties(II)* by C Pate, http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1087497/

Compare and contrast the two distributions.

You should discuss shape, centre and spread in relation to the context.

See marking schedule

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Extra paper if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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