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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

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SUPERVISOR'S USE ONLY

Level 2 Mathematics and Statistics, 2014

91267 Apply probability methods in solving problems

2.00 pm Wednesday 19 November 2014
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–14 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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As people grow older, the density of their bones decreases, making the bones more likely to fracture.

One way to prevent the fracture of bones is to take supplements.

A clinical trial was conducted to test a new supplement A.

A randomly selected group of 2127 people aged between 50 and 70 participated in a trial for the new supplement.

Approximately half of the people were given supplement A, and the rest were given a placebo (which looked the same but had no medical effect).

The results of the trial are shown in the following table.

	Number of people		
Treatment	**No fracture**	**At least one fracture**	**Total people**
Supplement A	973	92	1065
Placebo	914	148	1062
Total	1887	240	2127

(a) (i) What proportion of people in this trial had no fractures?

$\frac{1887}{2127} = 0.8872$

(ii) What proportion of people in this trial were given supplement A and had no fractures?

$\frac{973}{2127} = 0.4575$

(b) What percentage of people in this trial who did not have a fracture were given the placebo?

out of fracture

$\frac{914}{1887} \times 100 = 48.4\%$

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(c) A claim is made that taking supplement A halves the risk of having a bone fracture for people in this trial.

→ this goes on top of RR

Can this claim be justified statistically?

Support your answer with appropriate calculations.

P(fracture with supplement) = $\frac{92}{1065}$ = 0.086385

P(fracture with placebo) = $\frac{148}{1062}$ = 0.1393597 F

$\frac{0.086385}{0.1393597}$ = 0.6199

Therefore claim is incorrect its closer to 0.6 than 0.5

(d) Ngaire claims that taking supplement A slightly reduces the risk of having at least one fracture for people aged between 50 and 70. She says, in effect, that supplement A prevents one person in approximately 20 such people from having a fracture.

Is Ngaire's claim justified?

Use the results of the trial to support your answer, and include appropriate calculations.

If 100 people treated around 14 people on placebo would get a fracture. Around 9 on the supplement would. So this is 5 fewer.

$\frac{5}{100} = \frac{1}{20}$

So the claim is justified.

QUESTION TWO

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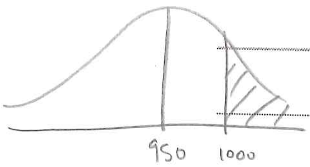
(a) Tests show that the bone mineral density (BMD) of 25-year-old females is approximately normally distributed, with a mean of 950 mg / cm^2 and a standard deviation of 125 mg / cm^2.

(i) What is the probability that a 25-year-old female has a BMD of more than 1000 mg / cm^2?



$z = \frac{1000 - 950}{125}$

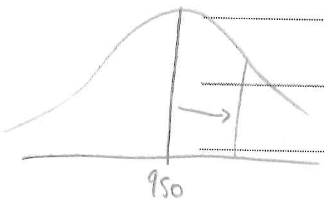
$= 0.4$

$P = 0.5 - 0.1554 = 0.3446$

(ii) Jane is a 25-year-old.

Her BMD has a z-value of 0.49

What is Jane's actual BMD measurement?



$0.49 = \frac{X - 950}{125}$

$X = 1011.25$ mg / cm^2

(iii) What proportion of 25-year-old females could be expected to have a BMD that is less than Jane's?

$\therefore P(X < 1011.25)$

look up 0.49 on tables

$= 0.1879 + 0.5$

$= 0.6879$

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(b) Tests show that the BMD of 25-year-old males is approximately normally distributed, with a standard deviation of 150 mg/cm^2.

If the probability that a randomly chosen 25-year-old male has a BMD above 1000 mg/cm^2 is 0.656, find the mean BMD of 25-year-old males.

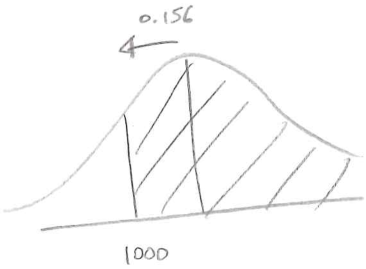
$$Z = \frac{X - \mu}{\sigma}$$

$$-0.401 = \frac{1000 - \mu}{150}$$

look this up backwards

$$-0.401 \times 150 - 1000 = -\mu$$

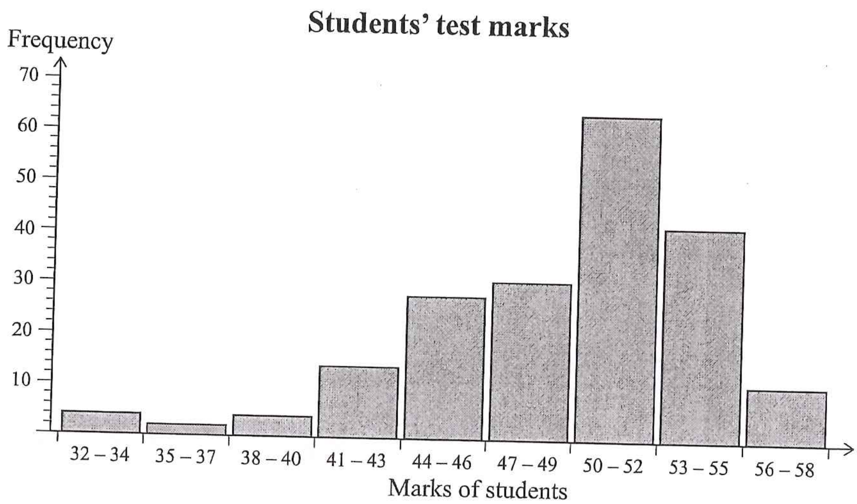
$\mu = 1060.15 \text{ mg cm}^{-2}$



(c) A test has questions of varying levels of difficulty.

Consider the graphs and tables shown on pages 6 and 7.

The graph below shows the distribution of the test marks of 200 students, and the table shows some related statistics.



Student marks	Frequency
32 – 34	4
35 – 37	2
38 – 40	4
41 – 43	14
44 – 46	28
47 – 49	31
50 – 52	64
53 – 55	42 117
56 – 58	11
Total	200

Statistics	Value
Mean	49.775
Minimum	32
Maximum	58
Range	26
Standard deviation	4.973

(i) What is the probability that a randomly chosen student has a mark less than 50?

$200 - 117 = 83$

$\frac{83}{200} = 0.415$

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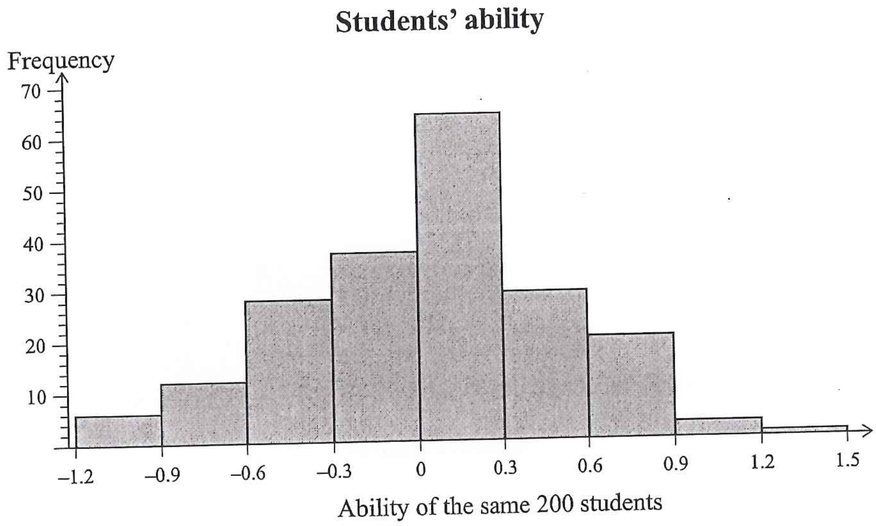
The graph below shows the distribution of the ability of the same students, and the table shows some related statistics.



Student ability	Frequency
–1.2 –	6
–0.9 –	12
–0.6 –	28
–0.3 –	37
0 –	64
0.3 –	29
0.6 –	20
0.9 –	3
1.2 –	1
Total	200

Statistics	Value
Mean	–0.1065
Minimum	–1.1
Maximum	1.3
Range	2.4
Standard deviation	0.465

(ii) How well does the test distinguish the ability of the most able students?

Justify your answer.

Defines top 4 students but the other 7 from the top category in c(i) it tells us little about

(iii) The graphs below have been copied from pages 6 and 7.

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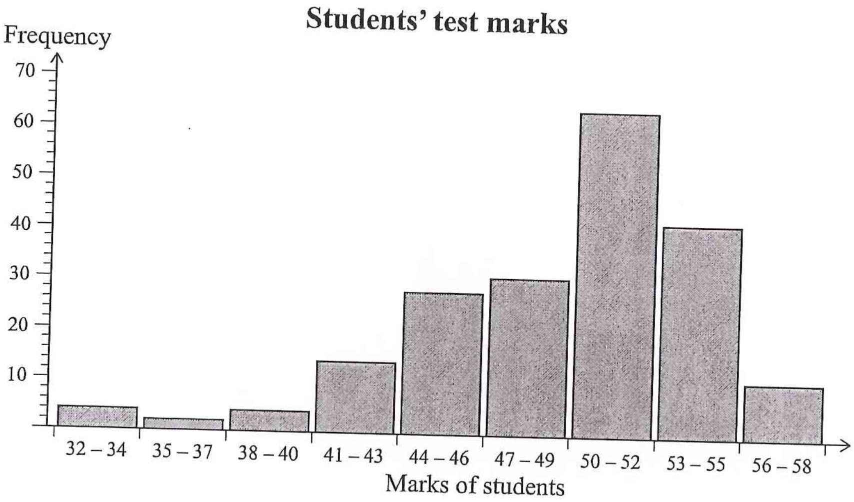
1



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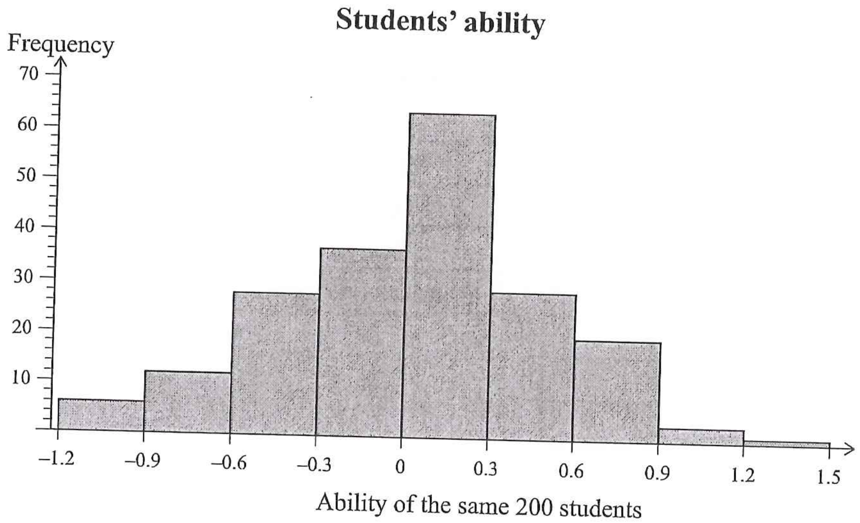

Discuss the distributions.

In your answer you should include some relevant calculations and some comparisons. You should also discuss shape, centre, and spread.

Shape 1 Skewed to the left
2 Normally distributed (bell shaped)

See marking schedule!

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QUESTION THREE

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Matiu and Whiti are playing a game with two fair six-sided dice numbered 1 to 6.

Matiu starts by rolling the two dice together.

If the sum of the numbers showing on the dice is 7 or 11, then Matiu wins the game.

If the sum of the numbers showing on the dice is 2, 3, or 12, then Matiu loses the game.

The table below shows the probability for each possible sum of the numbers showing on the dice.

Probabilities of the sum of the numbers showing on the dice

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
	L	L				W				W	L

(a) What is the probability that Matiu loses on the first roll?

$\frac{1}{36} + \frac{2}{36} + \frac{1}{36} = \frac{4}{36}$

(b) What is the probability that there is no winner on the first roll?

$\frac{3}{36} + \frac{4}{36} + \frac{5}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} = \frac{24}{36}$

or $\frac{2}{3}$

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(c) If neither Matiu nor Whiti wins the game on the first roll (the sum of the numbers showing on the dice is not 2, 3, 7, 11, or 12), then the actual sum of the numbers showing on the dice that Matiu has thrown on this first roll becomes Matiu's target score for the rest of that game.

Matiu keeps rolling the dice until either the sum on the dice is his target score, or he rolls a sum of 7, whichever occurs first. Whiti does not roll the dice.

If Matiu's target score occurs first, he wins; if a 7 occurs first, then Whiti wins.

Suppose Matiu gets a sum of 5 on the first roll.

(i) What is the probability Matiu will win the game on the second roll?

$P(5) = \frac{4}{36}$

ie he has to roll a 5 to win

(ii) What is the probability Matiu will win the game on the third roll?

Second roll and Third roll

P(not getting 5 or 7) × P(5)

$\frac{26}{36} \times \frac{4}{36} = \frac{13}{162}$

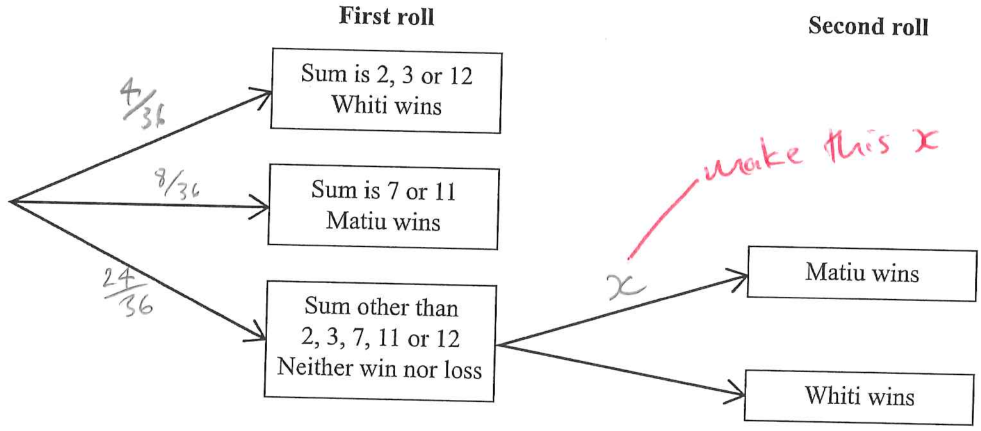
Question Three continues on the following page.

(d) Matiu and Whiti modify the game.

They want the modified game to:

- be fair
- finish after a maximum of two rolls.

The tree diagram below shows the outcomes for the modified game.



Find the probability that Matiu will win on the second roll.

P(win) = P(lose) ie both 0.5

Matiu wins   no one wins

$\therefore \frac{8}{36}$ or $\frac{24}{36}x = \frac{1}{2}$

$\frac{2}{9} + \frac{2x}{3} = 0.5$

$\frac{2x}{3} = \frac{5}{18}$

$x = \frac{5}{12}$

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