2

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91267

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NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Level 2 Mathematics and Statistics, 2015

91267 Apply probability methods in solving problems

2.00 p.m. Tuesday 10 November 2015
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Resource Sheet L2–MATHF.

Show ALL working.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–16 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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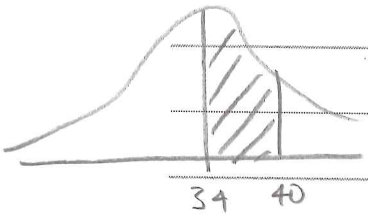
(a) The waiting time for a patient attending a medical centre before seeing a doctor is approximately normally distributed, with a mean of 34 minutes and a standard deviation of 8 minutes.

(i) Find the probability that a patient will wait between 34 and 40 minutes.



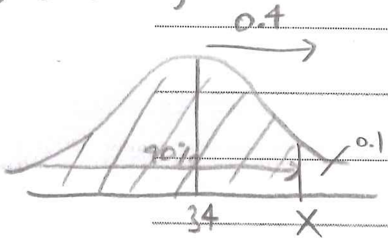
$z = \frac{40-34}{8}$

$= 0.75$

$P = 0.2734$

(ii) After how many minutes will 90% of patients have begun being seen by a doctor?

look up backwards



$z = \frac{X-34}{8}$

$1.281 = \frac{X-34}{8}$

$1.281 \times 8 + 34 = X$

$X = 44.25$ minutes

(iii) It is decided that waiting times must be changed so that at least 95% of patients will be seen by a doctor within 40 minutes.

Because of the administration required, the mean time cannot change, but it is known that for each doctor added to the duty teams, the standard deviation will reduce by 0.4 minutes.

How many doctors must be added to meet the new requirement?

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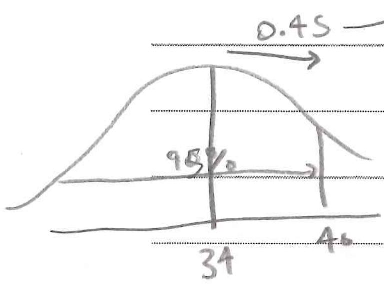

look up backwards

$1.645 = \frac{40-34}{\sigma}$

$1.645\sigma = 6$

$\sigma = \frac{6}{1.645}$

$\sigma = 3.647$

$\frac{8 - 3.647}{0.4} = 10.88$

∴ need another 11 doctors

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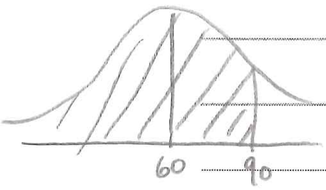
(b) At reception, patients are assessed on the urgency of their condition. This is done within two minutes of arrival.

It is thought that the waiting time before an assessment is done is approximately normally distributed with a mean of 60 seconds and standard deviation of 20 seconds.

(i) What proportion of patients would be assessed at reception within 90 seconds of arrival?



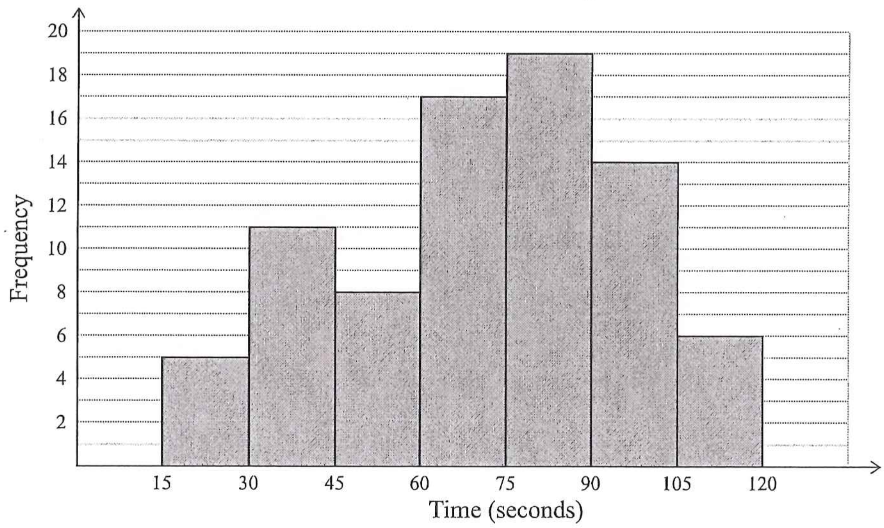
$z = \frac{90-60}{20}$

$= 1.5$

$P = 0.4332 + 0.5 = 0.9332$

(ii) A survey is carried out on 80 patients who arrive at reception. Patients are selected at random on a particular day. The results are shown in the frequency histogram below.

Graph (1)



What proportion of patients in the survey were assessed at reception within 90 seconds of arrival?

$80 - 14 - 6 = 60$

$\frac{60}{80} = 0.75$

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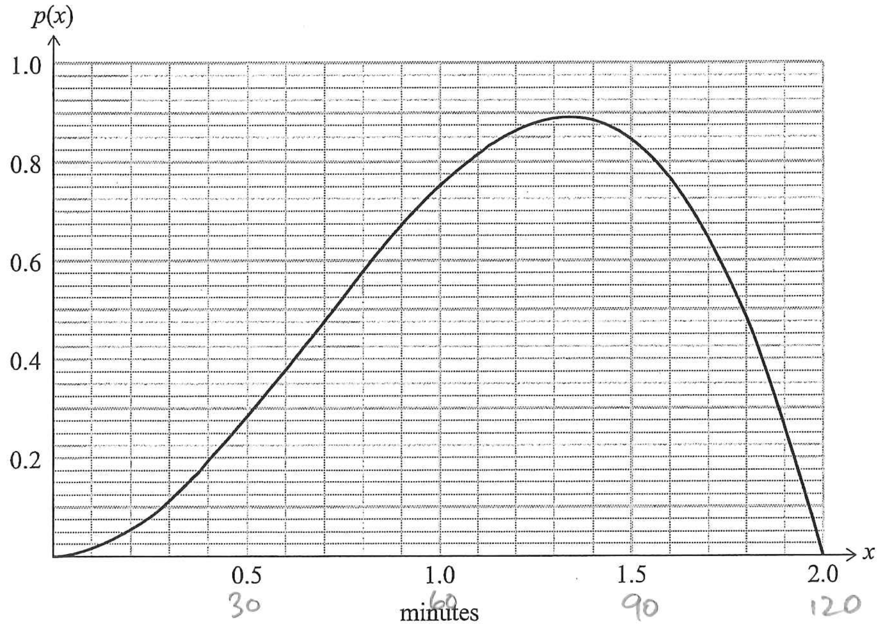
(iii) A statistician states that the assessment times are not normally distributed, but are more likely to approximate the distribution $p(x)$ below.

Graph ②



The associated probabilities (with minutes converted to seconds) are given in the following table:

Assessment Time (seconds)	0 –	15 –	30 –	45 –	60 –	75 –	90 –	105 – 120
Probability	0.01	0.05	0.10	0.16	0.21	0.22	0.17	0.08

Compare the frequency histogram for the survey of 80 patients with the distribution curve $p(x)$.

You should comment on the comparative shape, centre, and spread of the two distributions.

It is important to give numerical values to support your statements where possible.

→ two peaks

Graph ① Shape – not symmetrical, skewed to the left

② – also not symmetrical, skewed to the left

① Centre – mode 75-90, median 60-75

② Centre – median and mean to the right of centre

There is more space for your answer on the following page.

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Spread ① Range about 105 seconds
② Range of 2min (120 seconds)

①
② = Proportions similar apart from
30-60 ish

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QUESTION TWO

A study is conducted of 1500 randomly selected candidates for an international examination to investigate whether Year 12 candidates were as successful as those from Year 13.

The results are summarised in the table below:

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

(a) (i) What proportion of candidates in the study passed the examination?

$\frac{1200}{1500} = 0.8$

(ii) What proportion of candidates who failed the examination were from Year 12?

$\frac{33}{300} = 0.11$

(iii) There were about 52 500 candidates from Year 12 and Year 13 who attempted the examination.

Using the results of this study, how many candidates would be expected to be from Year 13, and pass the examination?

$\frac{853}{1500} = 0.5687$

$52500 \times 0.5687 = 29855$

so Yr 13 ontop for RR

(iv) It is claimed that Year 13 candidates are four times more likely to fail the examination than Year 12 candidates.

State whether or not you agree with this claim, showing full calculations to support your view.

Risk of Yr12 failing $\frac{33}{380} = 0.087$

Risk of Yr13 failing $\frac{267}{1120} = 0.238$

$RR = \frac{0.238}{0.087} = 2.74$

This is not close to 4, so I disagree with the claim.

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(b) The same study also considered the number of subjects the candidates were taking in their normal academic courses. It found that of the same sample of 1500 candidates, 682 were taking six subjects, while the rest were taking five subjects. Of the candidates who were taking five subjects, 192 failed the examination.

The table from page 7 is repeated here to help you answer the questions that follow.

	Year 12	Year 13	Total
Passed	347	853	1200
Failed	33	267	300
Total	380	1120	1500

	5 subjects	6 subjects	
Passed	626	574	1200
Failed	192	108	300
	818	682	1500

(i) What proportion of candidates in the study took six subjects and passed?

$\frac{574}{1500}$ = 0.3827

six on top for RR

(ii) On the evidence of this study, would you recommend that candidates take six subjects?

Support your answer with numerical calculations that consider the absolute and relative risks. You may also wish to comment on the sensibility of drawing any conclusions on this evidence.

Passed with 6 sub $= \frac{574}{682} = 0.8416$

Passed with 5 sub $= \frac{626}{818} = 0.7653$

$\frac{0.8416}{0.7653} = 1.0997$

∴ 10% more likely to pass if doing 6 subjects

BUT more able students are likely to be taking 6 subjects.

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