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Level 3 Mathematics and Statistics (Statistics), 2016

91585 Apply probability concepts in solving problems

2.00 p.m. Thursday 24 November 2016
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability concepts in solving problems.	Apply probability concepts, using relational thinking, in solving problems.	Apply probability concepts, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Show ALL working.

Make sure that you have the Formulae and Tables Booklet L3–STATF.

If you need more room for any answer, use the space provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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(a) A product demonstrator for a confectionary company approaches shoppers and asks them if they would like to taste the product. Of the last 528 female shoppers approached, 288 tasted the product. Of the last 31 male shoppers approached, 14 tasted the product.

Suppose one of these shoppers approached by the product demonstrator is selected at random.

(i) What is the probability that they tasted the product?

	♀	♂	
T	288	14	302
T'	240	17	257
	528	31	559

$P(\text{taste}) = \frac{302}{559}$

$= 0.54$

(ii) How many times as likely is it that a female shopper tasted the product compared to a male shopper?

Support your answer with appropriate statistical statements.

$P(\text{taste}/\text{female}) = \frac{288}{528} = 0.545$

$P(\text{taste}/\text{male}) = \frac{14}{31} = 0.452$

Relative risk $\frac{0.545}{0.452} = 1.2$ Females 1.2 times as likely to taste compared to male.

(iii) The product demonstrator claims that, in general, female shoppers are more likely to taste the product than male shoppers.

Discuss why the product demonstrator should be careful about using this data to make this claim.

- Only one sample - may get different results if in a different location or time of day.
- Male sample very small

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(b) At a particular supermarket, 43.4% of shoppers do not have a regular shopping day.

For shoppers who **do not** have a regular shopping day, 28.9% of these shoppers buy most of their groceries for the week on a weekend.

For shoppers who **do** have a regular shopping day, 41.2% of these shoppers buy most of their groceries for the week on a weekend.

(i) Without performing any additional calculations, explain why the events "has a regular shopping day" and "buys most of their groceries for the week on a weekend" are not independent.

P(Reg shopping) ≠ P(Not reg shopping)

They are not equal therefore they are not independent

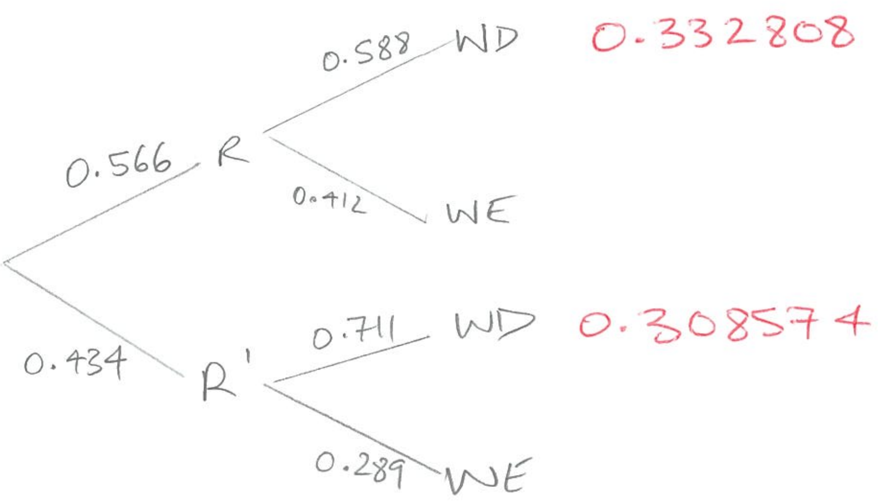
$0.289 \neq 0.412$

(ii) Suppose one of the shoppers at this supermarket is chosen at random.

Calculate the probability that this shopper buys most of their groceries for the week on a weekday.

$(0.566 \times 0.588) + (0.434 \times 0.711)$

$= 0.641$



(iii) Suppose three shoppers at this supermarket are chosen at random.

Calculate the probability that all three shoppers have a regular shopping day, and buy most of their groceries for the week on a weekend.

Support your answer with statistical statements and reasoning, including any assumption(s) made.

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$0.566 \times 0.412 = 0.233192$

$0.233192^3 = 0.0127$

Assumption is that
- all 3 shoppers are independent and their shopping/not shopping has to bearing on one another
- Sampling without replacement ie you can't pick the same person twice.

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QUESTION TWO

A market research company employs five different "observers" to watch how shoppers at a supermarket interact with products displayed on a particular shelf of an aisle of a supermarket. Each observer records the gender of the shopper, the shopper's estimated age band (e.g. 20 – 29 years), and whether the shopper stops to look at the products on this shelf.

(a) In the most recent study of shoppers at this supermarket, the market research company found that 42.1% of the shoppers observed stopped to look at the products displayed on this shelf. The company also found that 70.4% of the shoppers observed were female.

(i) One of the observers has used this information to predict that 17.1% of shoppers will be male and will not stop to look at products displayed on this shelf.

Show how the observer made this prediction, including stating any assumption(s) that were made.

P(male) = 1 − 0.704 = 0.296

P(not stopped) = 1 − 0.421 = 0.579

P(male ∩ not stopped) = 0.296 × 0.579

= 0.171 = 17.1%

Assumption is independence (add context)

(ii) It is also known that 38.7% of shoppers in the most recent study were female and stopped to look at the products displayed on this shelf.

Use this information to predict how many shoppers out of every 300 shoppers at the supermarket will be male and will not stop to look at products displayed on this shelf.

	♀	♂	
S	0.387	0.034	0.421
S'	0.317	0.262	0.579
	0.704	0.296	

P(male ∩ not stop) = 0.262

0.262 × 300 = 78.6

∴ around 79 shoppers will be male and not stop out of the next 300 shoppers.

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(b) Every 10th shopper observed in the recent study took a survey. One of the questions in this survey asked the shopper to select their actual age band. The market research company compared each shopper's estimated age band with their actual age band, and, based on these comparisons, calculated that each observer has an 86% accuracy rate for estimating the shopper's age band.

(i) Give ONE reason why this "accuracy" rate is only an estimate for the true probability that an "observer" will record each shopper's actual age band correctly.

- Based on a sample ie not a census
- Different observers will have different accuracy
- Accuracy could change with experience/confidence
- Accuracy could vary with gender/race/age etc

(ii) One of the observers has recorded the correct age band for 30 of the 42 shoppers they observed.

Discuss how carrying out a simulation would help the market research company consider whether this observer has a lower than 86% accuracy rate.

You do not need to design the simulation.

- A simulation would make the variation apparent
- Could compare to actual results

(c) Each observer also records whether each shopper has young children with them, buys any products on this shelf, and how long the shopper spends at the supermarket.

Of the 435 shoppers observed in the most recent study:

- 60 shoppers had young children with them, bought products on this shelf, and spent more than 30 minutes at the supermarket
- 86 shoppers had young children with them, and bought products on this shelf
- 62 shoppers bought products on this shelf, and spent more than 30 minutes at the supermarket
- 129 shoppers bought products on this shelf

- 32 shoppers had young children with them, but did not buy any products from this shelf, and did not spend more than 30 minutes at the supermarket
- 154 shoppers had young children with them
- 14 shoppers spent more than 30 minutes at the supermarket, but did not have young children with them, and did not buy products on this shelf.

A shopper from this study is selected at random.

Calculate the probability that the shopper did not have young children with them, did not buy any products on this shelf, and did not spend more than 30 minutes at the supermarket.

Support your answer with appropriate statistical statements or diagrams.

		B	B'	
YC	<30	86 − 60 26	32	154
	30+	60	36	
YC'	<30	41	224	281
	30+	62 − 60 2	14	
		129	306	435

$$P(YC' \cap <30 \cap B') = \frac{224}{435}$$

$$= 0.515$$

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QUESTION THREE

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(a) A certain supermarket has self-service checkout machines. Customers scan each item and then place the item into a shopping bag, which is then weighed by the machine. The machine then uses this weight to check that the item scanned is the same item that was placed in the shopping bag.

If the weight of the product stored on the machine (obtained from the barcode scanned) does not match the weight of the item measured by the machine, the machine flashes a red light; otherwise the machine flashes a green light. The self-service checkout machine does not always flash the correct coloured light.

In cases where the item scanned actually is the same item placed in the shopping bag, the machine will incorrectly flash a red light 3% of the time. In cases where the item scanned is not the same as the item placed in the shopping bag, the machine will correctly flash a red light 98% of the time.

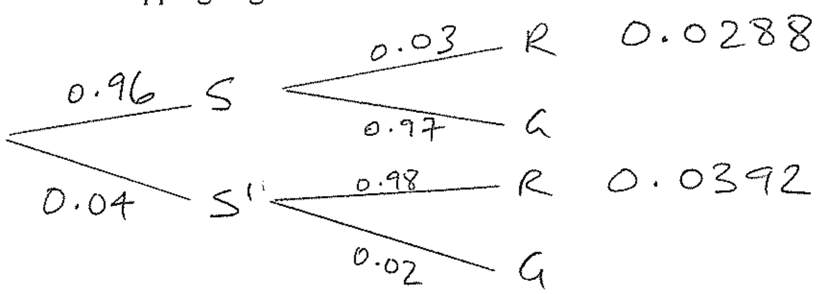
(i) Give ONE possible reason why the self-service checkout machine does not have a 100% accuracy rate when using item weights to check whether the correct item has been placed into the shopping bag.

- Machine error
- Product error – item scanned not what is put in bag.
- Weight variability – items won't all weigh exactly the same

(ii) At this supermarket, it is estimated that in 4% of the scans, the item scanned is not the same as the item placed in the shopping bag.

Suppose that an item is scanned and the machine flashes a red light.

Estimate the probability that the item scanned is not the same as the item placed in the shopping bag.



$$P(\text{not same} / \text{red}) = \frac{P(\text{not same} \cap \text{red})}{P(\text{red})}$$

$$= \frac{0.0392}{0.0392 + 0.0288}$$

$$= 0.576$$

(b) Each month, a supermarket sends customers who are part of its loyalty programme a $15 voucher if they have spent a minimum amount at the supermarket in the previous month.

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The supermarket has developed a model for how long it will take a customer to use the voucher after they receive it. Let X be the number of days after receiving a voucher that a customer uses the voucher.

A partially completed probability distribution table of the random variable X is shown below:

This table has been corrected from that used in the examination.

x	0	1	2	3	4	5	6	7	8	9	10 or more days
$P(X = x)$	0.012	0.019	0.038	0.056	0.076	0.098	0.073	0.047	0.018	0.031	0.532

(i) Under this model, estimate the probability that a customer uses the voucher no more than four days after receiving the voucher.

$P(X = 4) = 0.076$

$P(X \leq 4) = 0.076 + 0.056 + 0.038 + 0.019 + 0.012$

$= 0.201$

(ii) The supermarket is considering changing their loyalty programme. This change would require customers to use the voucher no more than 10 days **after the date of issue** (which is not the same date that the voucher is received by the customer).

Around 22% of the vouchers are received one day after the date of issue, around 35% of the vouchers are received two days after the date of issue, and the rest of the vouchers are received three days after the date of issue.

0.43

Use this information and the model developed by the supermarket to estimate the probability that a customer uses the voucher no more than10 days **after the date of issue**.

P(uses ≤9 days after) = 0.468

P(uses ≤8 days after) = 0.437

P(uses ≤7 days after) = 0.419

P(voucher used by expirey)

$= (0.468 \times 0.22) + (0.437 \times 0.35) + (0.419 \times 0.43)$

$= 0.436$

Question Three continues on the following page.

(iii) Discuss ONE limitation of the model you have used to obtain your probability estimate in part (ii).

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Assuming independence (put into context)

The new rule would likely change peoples plans & would no longer wait 10 days to use it

Assumes mail deliveries will be prompt, doesn't allow for longer delieuries