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Level 2 Mathematics and Statistics, 2019

91267 Apply probability methods in solving problems

9.30 a.m. Thursday 21 November 2019
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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QUESTION ONE

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Sean is interested in how heavy a student's schoolbag is. Using a random sample of New Zealand students, he obtained the weight of each student's schoolbag (to the nearest kg). The results of his investigation are shown below in Figure 1 and in Tables 1 and 2.

Figure 1: Weights of schoolbags of some NZ students

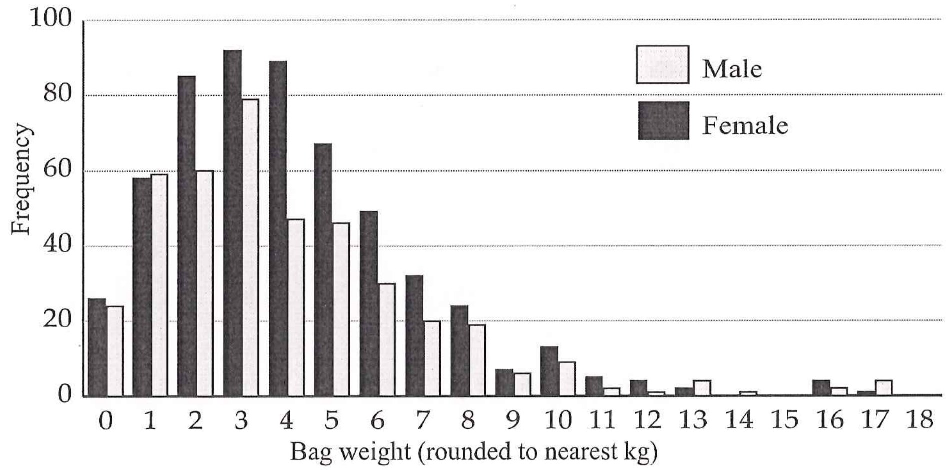

Table 1

Bag weight (kg)	Female students	Male students
0	26	24
1	58	59
2	85	60
3	92	79
4	89	47
5	67	46
6	49	30
7	32	20
8	24	19
9	7	6
10	13	9
11	5	2
12	4	1
13	2	4
14	0	1
15	0	0
16	4	2
17	1	4
18	0	0
Total	**558**	**413**

Table 2

Statistics for bag weight (kg)	Female students	Male students
Mean	4.1	4.0
Median	3.9	3.2

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(a) Use Table 1 to find the probability that a student randomly chosen from this sample:

(i) has a schoolbag weighing more than 10 kg.

$5+4+2+4+1+2+1+4+1+2+4=30$

$\frac{30}{971}=0.0309$ (4dp)

(ii) is male and has a schoolbag weighing 2 kg or less.

$\frac{60+59+24}{971}=\frac{143}{971}=0.1473$ (4dp)

(b) A "heavy" schoolbag is defined as one that weighs over 5 kg. Based on Sean's data, the following claim is made:

"Female students in New Zealand are more likely to have a 'heavy' schoolbag than male students."

Does Sean's data provide evidence to support this claim?

Include an appropriate relative risk calculation and other statistical considerations to support your answer.

Female 5kg+ $\frac{141}{558}=0.2527$ (4dp)

Male 5kg+ $\frac{98}{413}=0.2373$ (4dp)

$RR=\frac{0.2527}{0.2373}=1.06495$

Females slightly more likely to have a "heavy" bag. 6% more likely

6% is not a great deal more likely so may not be significant

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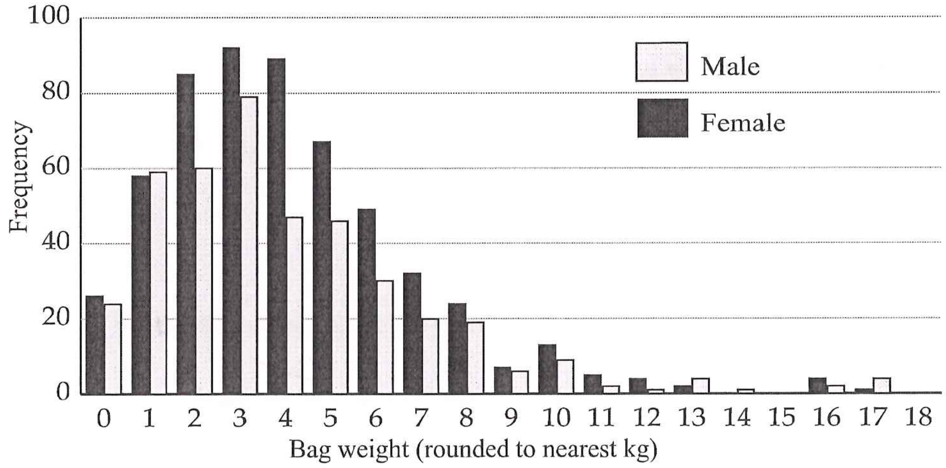
(c) Before doing this research, Sean expected that the schoolbag weights would be normally distributed.

(repeated from page 2)

Figure 1: Weights of schoolbags of some NZ students



(i) By referring to the shape of the distributions in Figure 1, describe clearly how these distributions are different from a normal distribution.

This graph is not symmetrical or bell shaped. There is a right skew.

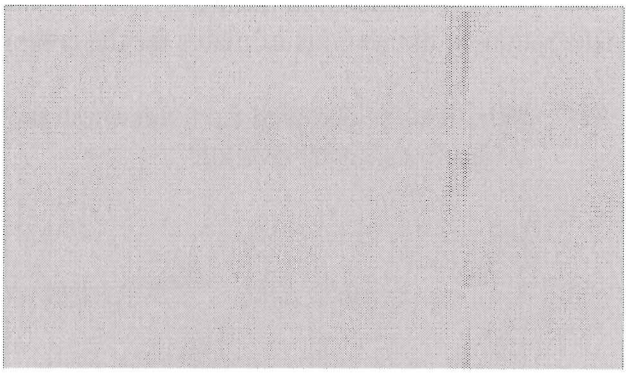
(ii) Give TWO reasons why it is **unlikely** that the schoolbag weights would be normally distributed.

see marking schedule

QUESTION TWO

A company produces hand-made plates.
The weights of the plates can be modelled by a normal distribution with a mean of 450 g and a standard deviation of 35 g.

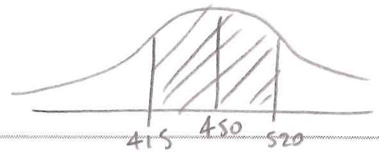
Working and/or diagrams must be shown.
Correct answer(s) alone will generally limit grades to Achievement.



Source: http://thehomescene.nz/beautiful-artisan-tableware-for-your-home/

(a) Find the probability that a randomly selected plate weighs:

(i) between 415 g and 520 g.



$z = \frac{x-\mu}{\sigma}$ & $z = \frac{520-450}{35}$

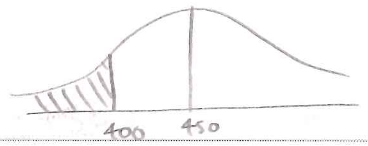
$z = \frac{415-450}{35}$ $= 2$

$= -1$

$P = 0.3413 + 0.4772$

$= 0.8186$

(ii) less than 400 g.



$z = \frac{400-450}{35}$

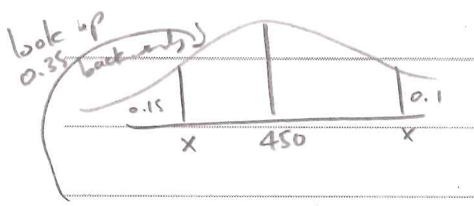
$= -1.4286$

$P = 0.5 - 0.4233$

$= 0.0767$

(b) The company rejects the heaviest 10% and the lightest 15% of all plates made.

What is the range of weights that the company accepts?



$1.281 = \frac{x-450}{35}$

$= 494.835$

$-1.036 = \frac{x-450}{35}$

$x = 413.74$

Between 413.74 and 494.835g

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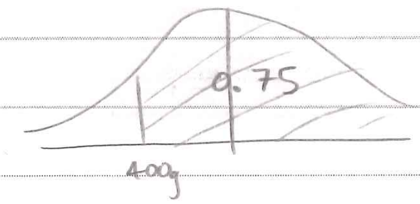
(c) Tayla makes plates at the company. Her plates also have a mean weight of 450 g. However, the standard deviation of the weights of the plates Tayla makes is higher than the standard deviation of the weights of plates for the company overall.

(i) What can be deduced from this about the way that Tayla makes her plates, compared with the company overall?

There is less variation in her product which means her plates are more consistent

(ii) For Tayla's plates, more than 75% weigh more than 400 g.

What is the **range** of possible values of the standard deviation of the weights of Tayla's plates?



$$-0.674 = \frac{400 - 450}{\sigma}$$

$$-0.674\sigma = -50$$

$$\sigma = 74.18$$

So the sd can be between 35 - 74.18g

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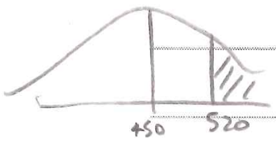
(d) Eddy also makes plates at the company. Two of his plates were randomly selected, and they both weighed over 520 g.

(i) Assume that the weights of Eddy's plates have the same normal distribution as the weights of the plates for the company overall.

What is the probability that 2 randomly selected plates made by Eddy would both weigh over 520 g?

$Z = \frac{520-450}{35}$



$= 2$

2 plates over 520 g

$= 0.0228 \times 0.0228$

$P = 0.5 - 0.4772$

$= 0.00051984$

$= 0.0228$

(ii) What could your answer to part (d)(i) suggest about the actual distribution of the weights of Eddy's plates, compared with the overall distribution for the company?

You must support your answer by using relevant probability calculations and / or diagrams.

① Possibly has a higher mean

② Larger variation

③ Both of the above

④ May not be normally distributed

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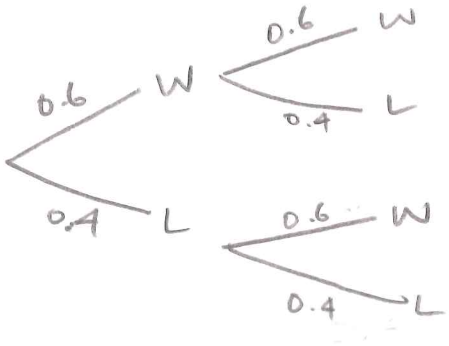
QUESTION THREE

At a school fund-raiser there is a dice game called Game A. The probability of winning Game A is 0.6.

(a) In "Dice-Twice", the player pays Ju-Eun 50c to take part, and plays Game A twice.

If the player wins both times, they receive \$2.

If the player wins only once, they receive \$1.



(i) What is the probability that the player receives \$2?

$P(\text{win}) = 0.6 \times 0.6 = 0.36$

(ii) What is the probability that the player receives \$1?

(0.6×0.4) or $(0.4 \times 0.6) = 0.48$

(iii) Kim decides to play "Dice-Twice" 100 times. Ju-Eun says that Kim "will profit by exactly \$110".

What are the **errors** in Ju-Eun's statement?

Support your answer with numerical calculations.

See marking schedule

(b) Tian invents a game and calls it "Two-In-Four". In his game, a player wins a prize if they can win twice at Game A. The player can keep playing Game A until

- they have won 2 games, or
- they have played a total of 4 games.

What is the probability of winning a prize in "Two-In-Four"?

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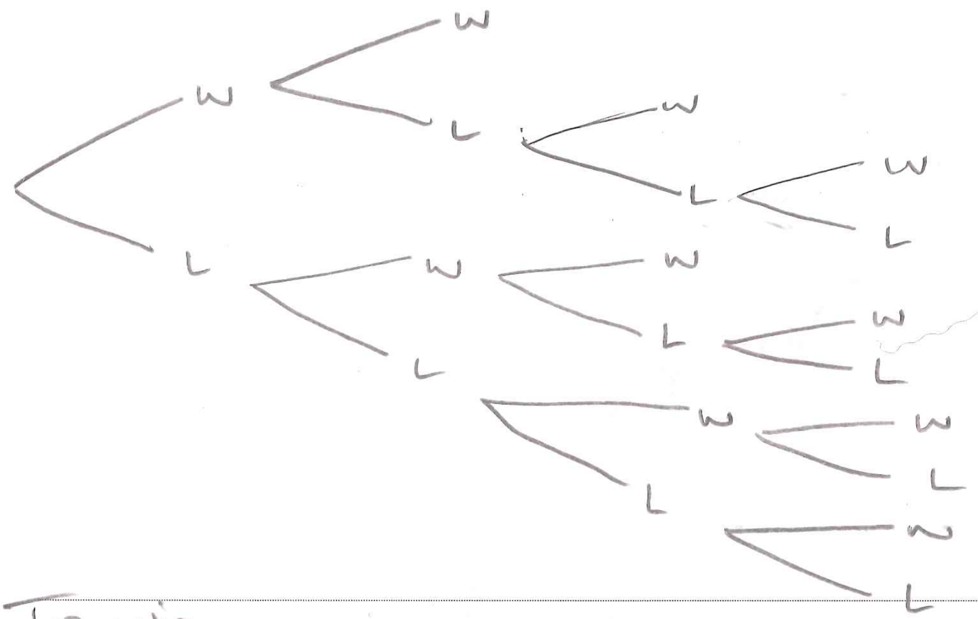


To win

WW = 0.6×0.6 = 0.36

WLW = 0.6×0.4×0.6 = 0.144

WLLW = 0.6×0.4×0.4×0.6 = 0.0576

LWW = 0.144

LWLW = 0.0576

LLWW = 0.0576

0.8208

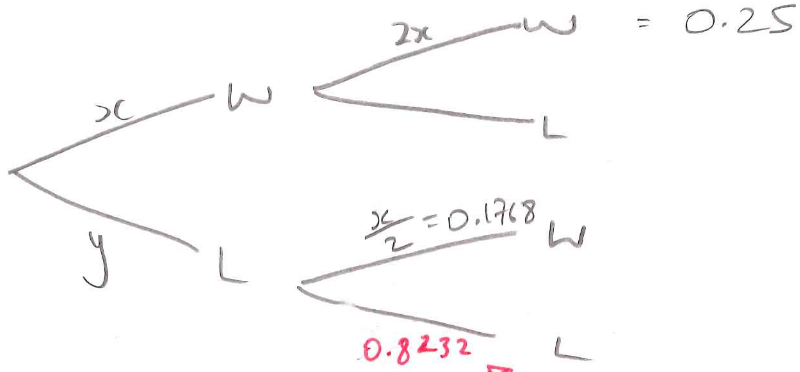
Question 3 continues on page 10 ➤

(c) Xuetao finds a different game that involves more skill. She plays this game twice.

- If she **wins** the first game, her probability of winning the second game is **twice** her probability of winning the first game.
- If she **loses** the first game, her probability of winning the second game is **half** her probability of winning the first game.

The probability that Xuetao wins one game or fewer is 0.75.

What is the probability that she loses both games?



$x \times 2x = 0.25$

$2x^2 = 0.25$

$x = \sqrt{(0.25 \div 2)}$

$= 0.3536$ Prob of winning

$y = 1 - 0.3536$

$= 0.6464$

Lose one and Lose second

$0.6464 \times 0.8232 = 0.5321$

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