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2

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Tick this box if there is no writing in this booklet

Level 2 Mathematics and Statistics 2020

91267 Apply probability methods in solving problems

9.30 a.m. Thursday 19 November 2020
Credits: Four

Achievement	Achievement with Merit	Achievement with Excellence
Apply probability methods in solving problems.	Apply probability methods, using relational thinking, in solving problems.	Apply probability methods, using extended abstract thinking, in solving problems.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should attempt ALL the questions in this booklet.

Make sure that you have Formulae Sheet L2–MATHF.

Show ALL working.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

TOTAL

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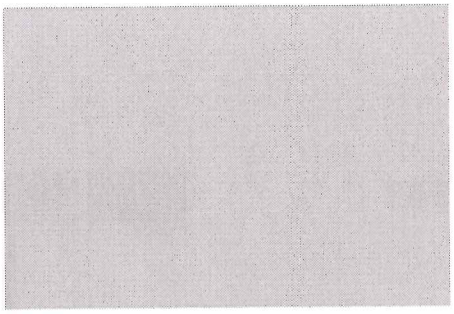
Owning an electric vehicle (EV) in New Zealand has become more popular as the technology has improved, prices have dropped, and people have become more aware of environmental issues.

The questions in this paper relate to research about EVs.



https://www.my-car-w.com/about

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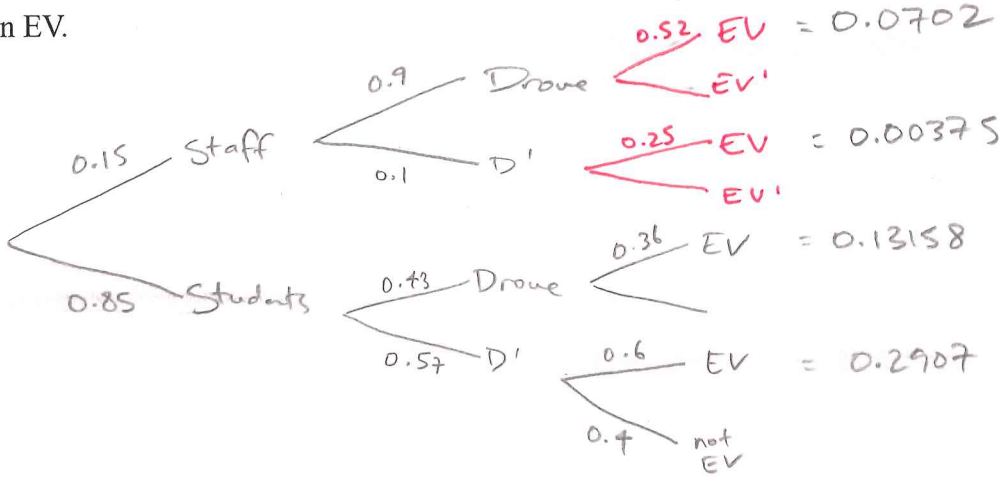
QUESTION ONE

Tia finds the results of a survey conducted on staff and students of a New Zealand university. The survey asked about various matters related to ownership of EVs.

- 15% of the survey respondents were staff and the rest were students.
- 90% of the staff and 43% of the students drove to the university.
- Of the students who drove to the university, 36% said they would like to own an EV, while 60% of the students who did not drive to the university said they would like to own an EV.



(a) (i) Find the probability that a randomly chosen respondent in the survey is a staff member who drove to the university.

P(Staff) × P(Drove)

0.15 × 0.9 = 0.135

(ii) Find the probability that a randomly chosen respondent in the survey is a student who did not drive to the university and would **not** like to own an EV.

P(Student) × P(Drive') × P(not EV)

0.85 × 0.57 × 0.4 = 0.1938

out of just the students.

(iii) If a **student** respondent said they would like to own an EV, what is the probability that they drove to the university?

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$$\frac{P(\text{Drove}) \times P(\text{EV})}{P(\text{EV})} =$$

$$\frac{(0.43 \times 0.36)}{(0.43 \times 0.36) + (0.57 \times 0.6)} = \frac{0.1548}{0.4968}$$

$$= 0.3116$$

(iv) Of the staff who drove to the university, 52% said they would like to own an EV, while only a quarter of the staff who didn't drive to the university would like to own an EV.

What percentage of **all** survey respondents would like to own an EV?

See tree

$$= 0.0702 + 0.00375 + 0.13158 + 0.2907$$

$$= 0.49623$$

$$\therefore 49.62\%$$

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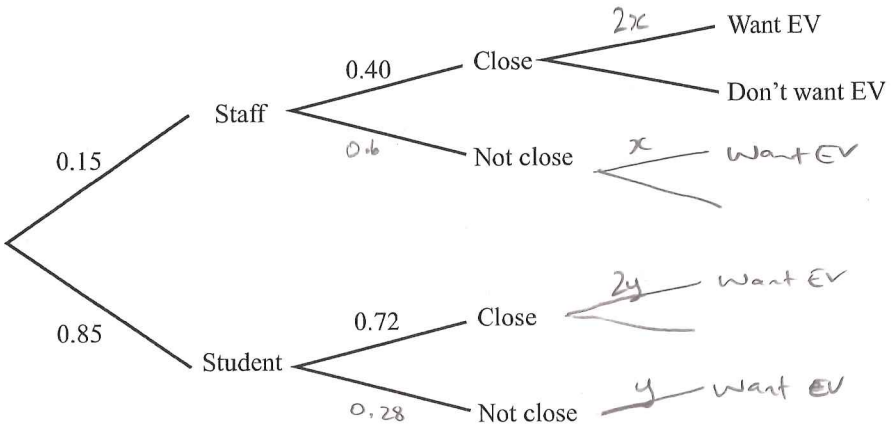
(b) Tia also learns that 40% of the staff and 72% of the students who responded to the survey live close to the university (travelled less than 15 km). She suspects that the distance they travel to the university would have an impact on their opinion on owning an EV.

Tia is told that both staff and students who live close to the university are **twice as likely** to want to own an EV as those who do not live close. She constructs a probability tree for this situation, part of which is shown below.



Tia has calculated that, overall, 49% of staff and 43% of students would like to own an EV (regardless of whether they drive to the university or not or how close they live).

Find out how much more likely it is for a randomly chosen survey respondent who lives close to university to want to own an EV than a respondent who does not live close.

Staff $0.4 \times 2x + 0.6 \times x = 0.49$

$0.8x + 0.6x = 0.49$

$1.4x = 0.49$

$x = 0.35$

Student $0.72 \times 2y + 0.28y = 0.43$

$1.72y = 0.43$

$y = 0.25$

Lives close

$0.15 \times 0.4 \times 0.7 + 0.85 \times 0.72 \times 0.5 = 0.348$

Not close

$0.15 \times 0.6 \times 0.35 + 0.85 \times 0.28 \times 0.25 = 0.091$

$\frac{0.348}{0.091}$

$= 3.82$

3.8 times more likely

QUESTION TWO

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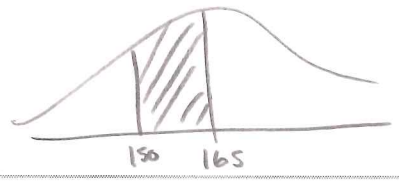
"Range anxiety" is the worry that electric vehicle (EV) drivers have about how long their battery charge will last. This is one of the most common concerns that stops people from buying an EV.

(a) A car manufacturer claimed that its EV was able to travel 172 km on one full battery charge. Geoff, an EV owner, recorded the distance that his car travelled after each full battery charge, and found it could be modelled by a normal distribution, with a mean of 165 km and a standard deviation of 14 km.

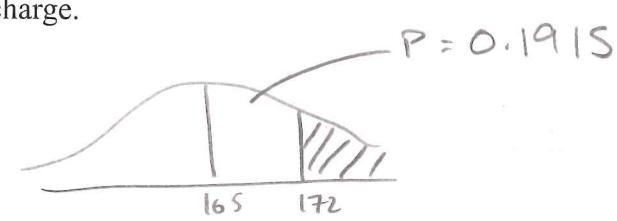
(i) Find the probability that Geoff's EV will travel between 150 km and 165 km on one full battery charge.



$z = \frac{x-\mu}{\sigma}$ $P(150 < x < 165) = 0.3579$

$= \frac{150-165}{14} = -1.0714$

(ii) Find the probability that Geoff's EV will travel more than 172 km on one full battery charge.



$z = \frac{172-165}{14}$

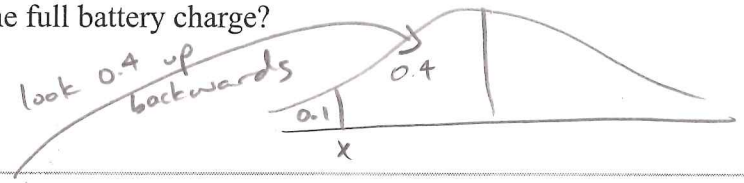
$= 0.5$

$0.5 - 0.1915$

$= 0.3085$

(iii) 10% of the time Geoff is unsatisfied with how far one full battery charge takes his EV.

For Geoff to be satisfied, what is the minimum distance his car will have to travel on one full battery charge?



$-1.281 = \frac{x-165}{14}$

$x = 147.066$

$\therefore$ minimum distance of 147.1 km

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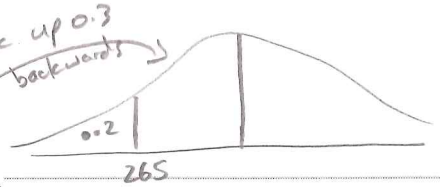
(b) The same car manufacturer released a second-generation EV.

A new owner of a second-generation EV found that 20% of the distances travelled on a full battery charge were less than 265 km.

Calculate an estimate for the mean distance travelled on a full charge by a second-generation EV for this owner. Assume that the distances travelled can be modelled by a normal distribution with a standard deviation of 14 km.

look up 0.3 backwards



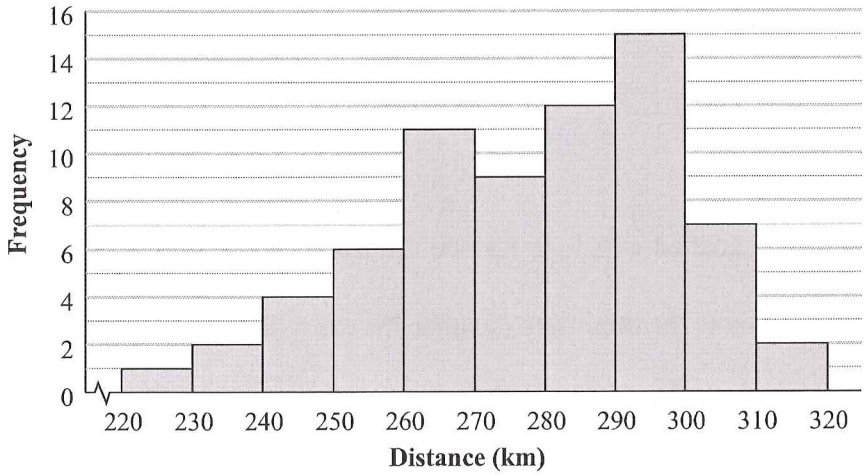
$$-0.841 = \frac{265 - \mu}{14}$$

$$-276.774 = -\mu$$

$$\mu = 276.774 \text{ km}$$

(c) An independent company completed 70 test drives of a second-generation EV. Each test started with a full battery charge and ended when an indicator showed that the expected remaining travel distance was less than 5 km. Each test involved driving on a range of city roads at different times of the day on weekdays. The results are shown in Figure 1 below.

Figure 1: Distance travelled in EV from one battery charge



The car manufacturer claimed that its second-generation EVs can travel 280 km on one battery charge on average **and** that these distances would be normally distributed, with a standard deviation of 14 km.

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Evaluate this claim by:

- comparing the shape, centre, and spread of the histogram in Figure 1 with the claimed normal distribution
- providing numerical values where appropriate
- considering the quality of the testing that led to the results reported in Figure 1.

See marking schedule

QUESTION THREE

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Studies have found some links between EV ownership and the installation of a home solar system.

An online survey was taken in North America of EV owners, plug-in hybrid vehicle (PHEV) owners and other interested people (who owned non-electric vehicles) during a three-month period in 2018.

The results are summarised in the table below.

Table 1: Vehicle type and home solar system ownership in North America

	Home solar system	No home solar system	Total
Electric vehicle (EV)	104	171	275
Plug-in hybrid electric vehicle (PHEV)	45	100	145
Non-electric vehicle	205	1375	1580
Total	354	1646	2000

(a) Use Table 1 to answer parts (i) to (iv).

(i) What proportion of survey respondents owned an EV or a PHEV?

$\frac{145+275}{2000} = \frac{420}{2000} = 0.21$

(ii) Which type of vehicle owner (EV, PHEV or non-electric) in North America is more likely to have a home solar system, according to these survey results?

EV $\frac{104}{275} = 0.3782$

PHEV $\frac{45}{145} = 0.3103$

Non $\frac{205}{1580} = 0.1297$

An EV owner is more likely to have a home solar system

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(iii) There were 22 144 home solar systems installed in New Zealand by the end of 2018.

Use the results from the North American study in Table 1 to estimate how many of the home solar system owners in New Zealand also have EVs.

$\frac{104}{354} = 0.2938$

$0.2938 \times 22144 = 6505.58$ (2dp)

∴ around 6506 would have solar & EV

(iv) Give at least one statistical reason why using Table 1 in your answer to part (iii) may not lead to a valid estimate in this case.

Sampling method (self selected online survey)

Can we apply findings from US to NZ

Please turn over ➤

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(b) A similar survey was conducted in Europe at the same time. Some of these results are shown in Table 2 below.

Table 2: Vehicle type and home solar system ownership in Europe

	Home solar system	No home solar system	Total
Electric vehicle (EV)	63	162	225
Plug-in hybrid electric vehicle (PHEV)	23	73	96
Non-electric vehicle	185	694	879
Total	271	929	1200

(i) Complete the table above and use it to find the probability that a randomly selected respondent whose vehicle was non-electric, had a home solar system.

$\frac{185}{879} = 0.2105$ (4 dp)

(ii) A newspaper article quoted some of the findings from the two studies shown in Table 1 and Table 2. It claimed that:

- In North America, EV owners are **3 times** as likely to have a home solar system as non-electric vehicle owners
- In Europe, EV owners are **only 30%** more likely to have a home solar system than non-electric vehicle owners.

(Table 1 from page 8 is repeated below)

Table 1: Vehicle type and home solar system ownership in North America

	Home solar system	No home solar system	Total
Electric vehicle (EV)	104	171	275
Plug-in hybrid electric vehicle (PHEV)	45	100	145
Non-electric vehicle	205	1375	1580
Total	354	1646	2000

Can these claims be supported?

Include probability calculations in your answer, and comment on the validity of drawing these conclusions, based on the evidence given in Question Three.

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North America

$\frac{\text{EV \& solar}}{\text{EV}} = \frac{104}{275} = 0.3782$

$\frac{\text{no EV \& solar}}{\text{no EV}} = \frac{205}{1580} = 0.1297$

$0.1297 \times 3 = 0.3892$

This is not quite 0.3782 so not 3x as likely

or RR $\frac{0.3782}{0.1297} = 2.916$ not quite 3

Europe

$\frac{\text{EV \& solar}}{\text{EV}} = \frac{63}{225} = 0.28$

$\frac{\text{no EV \& solar}}{\text{no EV}} = \frac{185}{879} = 0.2104$

$RR = \frac{0.28}{0.2104} = 1.33$

Yes 33% more likely to have solar if EV owner

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

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Acknowledgements
Material from the following sources has been adapted for use in this examination:

https://cleantechnica.com/2019/02/16/electric-car-ownerships-connection-to-rooftop-solar-energy-conservation/
https://cleantechnica.com/2017/06/25/28-40-ev-drivers-solar-panels-cleantechnica-ev-report/